Solution

Fig. 7

As regards the text of the problem, the sentence “The same quantities of heat have been supplied to both balls.” is not too clear. We will follow intuitive understanding of this sentence, i.e. we will assume that both systems (A – the hanging ball and B – the ball resting on the plane) received the same portion of energy from outside. One should realize, however, that it is not the only possible interpretation.

When the balls are warmed up, their mass centers are moving as the radii of the balls are changing. The mass center of the ball A goes down, while the mass center of the ball B goes up. It is shown in Fig. 7 (scale is not conserved).

Displacement of the mass center corresponds to a change of the potential energy of the ball in the gravitational field.

In case of the ball A the potential energy decreases. From the 1st principle of thermodynamics it corresponds to additional heating of the ball.

In case of the ball B the potential energy increases. From the 1st principle of thermodynamics it corresponds to some “losses of the heat provided” for performing a mechanical work necessary to rise the ball. The net result is that the final temperature of the ball B should be lower than the final temperature of the ball A.

The above effect is very small. For example, one may find (see later) that for balls made of lead, with radius 10 cm, and portion of heat equal to 50 kcal, the difference of the final temperatures of the balls is of order 10^{-5} K. For spatial and time fluctuations such small quantity practically cannot be measured.

Calculation of the difference of the final temperatures was not required from the participants. Nevertheless, we present it here as an element of discussion.

We may assume that the work against the atmospheric pressure can be neglected. It is obvious that this work is small. Moreover, it is almost the same for both balls. So, it should not affect the difference of the temperatures substantially. We will assume that such quantities as specific heat of lead and coefficient of thermal expansion of lead are constant (i.e. do not depend on temperature).

The heat used for changing the temperatures of balls may be written as

$$Q_i = mc\Delta t_i, \quad \text{where } i = A \text{ or } B,$$

Here: m denotes the mass of ball, c - the specific heat of lead and Δt_i - the change of the temperature of ball.

The changes of the potential energy of the balls are (neglecting signs):

$$\Delta E_i = mgr\alpha\Delta t_i, \quad \text{where } i = A \text{ or } B.$$

Here: g denotes the gravitational acceleration, r - initial radius of the ball, α - coefficient of thermal expansion of lead. We assume here that the thread does not change its length.

Taking into account conditions described in the text of the problem and the interpretation mentioned at the beginning of the solution, we may write:

$$Q = Q_A - A\Delta E_A, \quad \text{for the ball } A,$$
$$Q = Q_B + A\Delta E_B, \quad \text{for the ball } B.$$

A denotes the thermal equivalent of work: $A \approx 0.24\frac{\text{cal}}{\text{J}}$. In fact, A is only a conversion ratio between calories and joules. If you use a system of units in which calories are not present, you may omit A at all.

Thus

$$Q = (mc - Amgr\alpha)\Delta t_A, \quad \text{for the ball } A,$$
$$Q = (mc + Amgr\alpha)\Delta t_B, \quad \text{for the ball } B$$

and

$$\Delta t_A = \frac{Q}{mc - Amgr\alpha}, \qquad \Delta t_B = \frac{Q}{mc + Amgr\alpha}.$$

Finally we get

$$\Delta t = \Delta t_A - \Delta t_B = \frac{2Agr\alpha}{c^2 - (Agr\alpha)^2}\frac{Q}{m} \approx \frac{2AQgr\alpha}{mc^2}.$$

(We neglected the term with α^2 as the coefficient α is very small.)

Now we may put the numerical values: $Q = 50$ kcal, $A \approx 0.24$ cal/J, $g \approx 9.8$ m/s^2, $m \approx 47$ kg (mass of the lead ball with radius equal to 10 cm), $r = 0.1$ m, $c \approx 0.031$ cal/(g·K), $\alpha \approx 29 \cdot 10^{-6}$ K^{-1}. After calculations we get $\Delta t \approx 1.5 \cdot 10^{-5}$ K.