Solution

If the cord is stressed the cylinder and the block are moving with the same acceleration *a*. Let *F* be the tension in the cord, *S* the frictional force between the cylinder and the inclined plane (*Fig. 2*). The angular acceleration of the cylinder is *a*/*r*. The net force causing the acceleration of the block:

$$m_2 a = m_2 g \sin\alpha - \mu m_2 g \cos\alpha + F ,$$

and the net force causing the acceleration of the cylinder:

$$m_1 a = m_1 g \sin\alpha - S - F .$$

The equation of motion for the rotation of the cylinder:

$$S\, r = \frac{a}{r} \cdot I .$$

(*I* is the moment of inertia of the cylinder, *S*·*r* is the torque of the frictional force.)
Solving the system of equations we get:

$$a = g \cdot \frac{(m_1 + m_2)\sin\alpha - \mu m_2 \cos\alpha}{m_1 + m_2 + \frac{I}{r^2}} , \tag{1}$$

$$S = \frac{I}{r^2} \cdot g \cdot \frac{(m_1 + m_2)\sin\alpha - \mu m_2 \cos\alpha}{m_1 + m_2 + \frac{I}{r^2}} , \tag{2}$$

$$F = m_2 g \cdot \frac{\mu\left(m_1 + \frac{I}{r^2}\right)\cos\alpha - \frac{I \sin\alpha}{r^2}}{m_1 + m_2 + \frac{I}{r^2}} . \tag{3}$$

The moment of inertia of a solid cylinder is $I = \frac{m_1 r^2}{2}$. Using the given numerical values:

$$a = g \cdot \frac{(m_1 + m_2)\sin\alpha - \mu m_2 \cos\alpha}{1.5 m_1 + m_2} = 0.3317 g = \mathbf{3.25\ m/s^2} ,$$

$$S = \frac{m_1 g}{2} \cdot \frac{(m_1 + m_2)\sin\alpha - \mu m_2 \cos\alpha}{1.5 m_1 + m_2} = \mathbf{13.01\ N} ,$$

$$F = m_2 g \cdot \frac{(1.5\mu \cos\alpha - 0.5 \sin\alpha) m_1}{1.5 m_1 + m_2} = \mathbf{0.192\ N} .$$

Discussion (See *Fig. 3*.)

The condition for the system to start moving is $a > 0$. Inserting $a = 0$ into (1) we obtain the limit for angle α_1:

$$\tan\alpha_1 = \mu \cdot \frac{m_2}{m_1 + m_2} = \frac{\mu}{3} = 0.0667 , \quad \alpha_1 = 3.81° .$$

For the cylinder separately $\alpha_1 = 0$, and for the block separately $\alpha_1 = \tan^{-1}\mu = 11.31°$.

If the cord is not stretched the bodies move separately. We obtain the limit by inserting $F = 0$ into (3):

$$\tan\alpha_2 = \mu \cdot \left(1 + \frac{m_1 r^2}{I}\right) = 3\mu = 0.6, \quad \alpha_2 = 30.96°.$$

The condition for the cylinder to slip is that the value of S (calculated from (2) taking the same coefficient of friction) exceeds the value of $\mu m_1 g\cos\alpha$. This gives the same value for α_3 as we had for α_2. The acceleration of the centers of the cylinder and the block is the same: $g(\sin\alpha - \mu\cos\alpha)$, the frictional force at the bottom of the cylinder is $\mu m_1 g\cos\alpha$, the peripheral acceleration of the cylinder is $\mu \cdot \frac{m_1 r^2}{I} \cdot g\cos\alpha$.

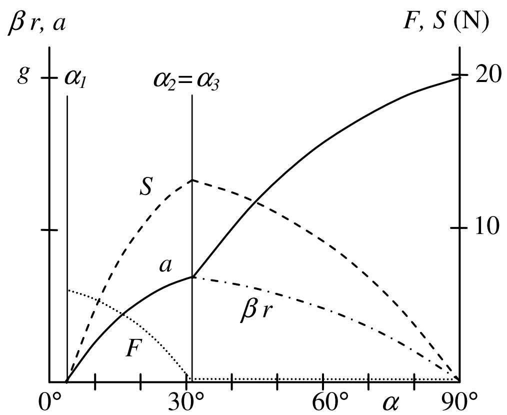

Figure 3