Solution

If the volume at temperature t_1 is V_1, then the volume at temperature 0°C is $V_{10} = V_1/(1+\beta t_1)$. In the same way if the volume at t_2 temperature is V_2, at 0°C we have $V_{20} = V_2/(1+\beta t_2)$. Furthermore if the density of the liquid at 0°C is d, then the masses are $m_1 = V_{10}d$ and $m_2 = V_{20}d$, respectively. After mixing the liquids the temperature is

$$t = \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2}.$$

The volumes at this temperature are $V_{10}(1+\beta t)$ and $V_{20}(1+\beta t)$.
The sum of the volumes after mixing:

$$V_{10}(1+\beta t) + V_{20}(1+\beta t) = V_{10} + V_{20} + \beta(V_{10} + V_{20})t =$$

$$= V_{10} + V_{20} + \beta \cdot \frac{m_1 + m_2}{d} \cdot \frac{m_1 t_1 + m_2 t_2}{m_1 + m_2} =$$

$$= V_{10} + V_{20} + \beta\left(\frac{m_1 t_1}{d} + \frac{m_2 t_2}{d}\right) = V_{10} + \beta V_{10} t_1 + V_{20} + \beta V_{20} t_2 =$$

$$= V_{10}(1+\beta t_1) + V_{20}(1+\beta t_2) = V_1 + V_2$$

The sum of the volumes is constant. In our case it is 410 cm^3. The result is valid for any number of quantities of toluene, as the mixing can be done successively adding always one more glass of liquid to the mixture.