

Solution:

We use the following notation:

t	temperature of the final equilibrium state,
$t_0 = 0^\circ\text{C}$	the melting point of ice under normal pressure conditions,
M_2	final mass of water,
M_3	final mass of ice,
$m'_2 \leq m_2$	mass of water, which freezes to ice,
$m'_3 \leq m_3$	mass of ice, which melts to water.

a) Generally, four possible processes and corresponding equilibrium states can occur:

1. $t_0 < t < t_2$, $m'_2 = 0$, $m'_3 = m_3$, $M_2 = m_2 + m_3$, $M_3 = 0$.

Unknown final temperature t can be determined from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t) = m_3c_3(t_0 - t_3) + m_3l + m_3c_2(t - t_0). \quad (7)$$

However, only the solution satisfying the condition $t_0 < t < t_2$ does make physical sense.

2. $t_3 < t < t_0$, $m'_2 = m_2$, $m'_3 = 0$, $M_2 = 0$, $M_3 = m_2 + m_3$.

Unknown final temperature t can be determined from the equation

$$m_1c_1(t_2 - t) + m_2c_2(t_2 - t_0) + m_2l + m_2c_3(t_0 - t) = m_3c_3(t - t_3). \quad (8)$$

However, only the solution satisfying the condition $t_3 < t < t_0$ does make physical sense.

3. $t = t_0$, $m'_2 = 0$, $0 \leq m'_3 \leq m_3$, $M_2 = m_2 + m'_3$, $M_3 = m_3 - m'_3$.

Unknown mass m'_3 can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) = m_3c_3(t - t_3) + m'_3l. \quad (9)$$

However, only the solution satisfying the condition $0 \leq m'_3 \leq m_3$ does make physical sense.

4. $t = t_0$, $0 \leq m'_2 \leq m_2$, $m'_3 = 0$, $M_2 = m_2 - m'_2$, $M_3 = m_3 + m'_2$.

Unknown mass m'_2 can be calculated from the equation

$$(m_1c_1 + m_2c_2)(t_2 - t_0) + m'_2l = m_3c_3(t_0 - t_3). \quad (10)$$

However, only the solution satisfying the condition $0 \leq m'_2 \leq m_2$ does make physical sense.

b) Substituting the particular values of m_1 , m_2 , m_3 , t_2 and t_3 to equations (7), (8) and (9) one obtains solutions not making the physical sense (not satisfying the above conditions for t , respectively m'_3). The real physical process under given conditions is given by the equation (10) which yields

$$m'_2 = \frac{m_3c_3(t_0 - t_3) - (m_1c_1 + m_2c_2)(t_2 - t_0)}{l}.$$

Substituting given numerical values one gets $m'_2 = 0.11$ kg. Hence, $t = 0^\circ\text{C}$, $M_2 = m_2 - m'_2 = 0.89$ kg, $M_3 = m_3 + m'_2 = 2.11$ kg.