

Solution:

Condition for the maximum reinforcement can be written as

$$2dn - \frac{\lambda_k}{2} = k\lambda_k, \text{ for } k = 0, 1, 2, \dots,$$

i.e.

$$2dn = (2k + 1)\frac{\lambda_k}{2}, \quad (13)$$

with d being thickness of the layer, n the refractive index and k maximum order. Let us denote $\lambda' = 1150$ nm. Since for $\lambda = 400$ nm the condition for maximum is satisfied by the assumption, let us denote $\lambda_p = 400$ nm, where p is an unknown integer identifying the maximum order, for which

$$\lambda_p(2p + 1) = 4dn \quad (14)$$

holds true. The equation (13) yields that for fixed d the wavelength λ_k increases with decreasing maximum order k and vice versa. According to the

assumption,

$$\lambda_{p-1} < \lambda' < \lambda_{p-2},$$

i.e.

$$\frac{4dn}{2(p-1)+1} < \lambda' < \frac{4dn}{2(p-2)+1}.$$

Substituting to the last inequalities for $4dn$ using (14) one gets

$$\frac{\lambda_p(2p+1)}{2(p-1)+1} < \lambda' < \frac{\lambda_p(2p+1)}{2(p-2)+1}.$$

Let us first investigate the first inequality, straightforward calculations give us gradually

$$\lambda_p(2p+1) < \lambda'(2p-1), \quad 2p(\lambda' - \lambda_p) > \lambda' + \lambda_p,$$

i.e.

$$p > \frac{1}{2} \frac{\lambda' + \lambda_p}{\lambda' - \lambda_p} = \frac{1}{2} \frac{1150 + 400}{1150 - 400} = 1. \dots \quad (15)$$

Similarly, from the second inequality we have

$$\lambda_p(2p+1) > \lambda'(2p-3), \quad 2p(\lambda' - \lambda_p) < 3\lambda' + \lambda_p,$$

i.e.

$$p < \frac{1}{2} \frac{3\lambda' + \lambda_p}{\lambda' - \lambda_p} = \frac{1}{2} \frac{3 \cdot 1150 + 400}{1150 - 400} = 2. \dots \quad (16)$$

The only integer p satisfying both (15) and (16) is $p = 2$.

Let us now find the thickness d of the air layer:

$$d = \frac{\lambda_p}{4}(2p+1) = \frac{400}{4}(2 \cdot 2 + 1) \text{ nm} = 500 \text{ nm}.$$

Substituting d to the equation (13) we can calculate λ_{p-1} , *i.e.* λ_1 :

$$\lambda_1 = \frac{4dn}{2(p-1)+1} = \frac{4dn}{2p-1}.$$

Introducing the particular values we obtain

$$\lambda_1 = \frac{4 \cdot 500 \cdot 1}{2 \cdot 2 - 1} \text{ nm} = 666.7 \text{ nm}.$$

Finally, let us determine temperature growth Δt . Generally, $\Delta l = \alpha l \Delta t$ holds true. Denoting the cube edge by h we arrive at $d = \alpha h \Delta t$. Hence

$$\Delta t = \frac{d}{\alpha h} = \frac{5 \cdot 10^{-7}}{8 \cdot 10^{-6} \cdot 2 \cdot 10^{-2}} \text{ }^\circ\text{C} = 3.1 \text{ }^\circ\text{C}.$$