Question 1.

The blocks slide relative to the prism with accelerations $\mathbf{a}_1$ and $\mathbf{a}_2$, which are parallel to its sides and have the same magnitude a (see Fig. 1.1). The blocks move relative to the earth with accelerations:

(1.1) $\mathbf{w}_1 = \mathbf{a}_1 + \mathbf{a}_0$;

(1.2) $\mathbf{w}_2 = \mathbf{a}_2 + \mathbf{a}_0$.

Now we project $\mathbf{w}_1$ and $\mathbf{w}_2$ along the x- and y-axes:

(1.3) $w_{1x} = a\cos\alpha_1 - a_0$;

(1.4) $w_{1y} = a\sin\alpha_1$;

(1.5) $w_{2x} = a\cos\alpha_2 - a_0$;

(1.6) $w_{2y} = -a\sin\alpha_2$.

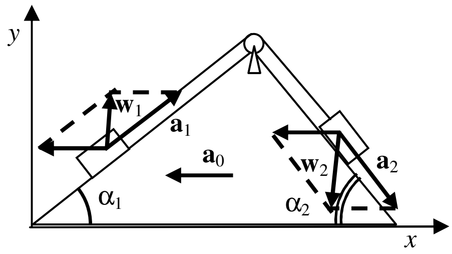

Fig. 1.1

The equations of motion for the blocks and for the prism have the following vector forms (see Fig. 1.2):

(1.7) $m_1\mathbf{w}_1 = m_1\mathbf{g} + \mathbf{R}_1 + \mathbf{T}_1$;

(1.8) $m_2\mathbf{w}_2 = m_2\mathbf{g} + \mathbf{R}_2 + \mathbf{T}_2$;

(1.9) $M\mathbf{a}_0 = M\mathbf{g} - \mathbf{R}_1 - \mathbf{R}_2 + \mathbf{R} - \mathbf{T}_1 - \mathbf{T}_2$.

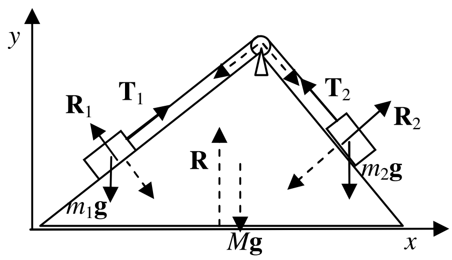

Fig. 1.2

The forces of tension $\mathbf{T}_1$ and $\mathbf{T}_2$ at the ends of the thread are of the same magnitude T since the masses of the thread and that of the pulley are negligible. Note that in equation (1.9) we account for the net force $-(\mathbf{T}_1 + \mathbf{T}_2)$, which the bended thread exerts on the prism through the pulley. The equations of motion result in a system of six scalar equations when projected along x and y:

(1.10) $m_1 a\cos\alpha_1 - m_1 a_0 = T\cos\alpha_1 - R_1\sin\alpha_1$;

(1.11) $m_1 a\sin\alpha_1 = T\sin\alpha_1 + R_1\cos\alpha_1 - m_1 g$;

(1.12) $m_2 a\cos\alpha_2 - m_2 a_0 = -T\cos\alpha_2 + R_2\sin\alpha_2$;

(1.13) $m_2 a\sin\alpha_2 = T\sin\alpha_2 + R_2\sin\alpha_2 - m_2 g$;

(1.14) $-Ma_0 = R_1\sin\alpha_1 - R_2\sin\alpha_2 - T\cos\alpha_1 + T\cos\alpha_2$;

(1.15) $0 = R - R_1\cos\alpha_1 - R_2\cos\alpha_2 - Mg$.

By adding up equations (1.10), (1.12), and (1.14) all forces internal to the system cancel each other. In this way we obtain the required relation between accelerations a and a_0:

(1.16) $$a = a_0 \frac{M + m_1 + m_2}{m_1 \cos\alpha_1 + m_2 \cos\alpha_2}.$$

The straightforward elimination of the unknown forces gives the final answer for a_0:

(1.17) $$a_0 = \frac{(m_1 \sin\alpha_1 - m_2 \sin\alpha_2)(m_1 \cos\alpha_1 + m_2 \cos\alpha_2)}{(m_1 + m_2 + M)(m_1 + m_2) - (m_1 \cos\alpha_1 + m_2 \cos\alpha_2)^2}.$$

It follows from equation (1.17) that the prism will be in equilibrium ($a_0 = 0$) if:

(1.18) $$\frac{m_1}{m_2} = \frac{\sin\alpha_2}{\sin\alpha_1}.$$