***Solution***

According to the Bohr model the energy levels of the hydrogen atom are given by the formula:

$$E_n = -\frac{E_i}{n^2},$$

where $n =$ 1, 2, 3, … The ground state corresponds to $n = 1$, while the lowest excited state corresponds to $n = 2$. Thus, the smallest energy necessary for excitation of the hydrogen atom is:

$$\Delta E = E_2 - E_1 = E_i(1 - \tfrac{1}{4}) = \tfrac{3}{4}E_i .$$

During an inelastic collision a part of kinetic energy of the colliding particles is converted into their internal energy. The internal energy of the system of two hydrogen atoms considered in the problem cannot be changed by less than $\Delta E$. It means that if the kinetic energy of the colliding atoms with respect to their center of mass is less than $\Delta E$, then the collision must be an elastic one. The value of $v_0$ can be found by considering the critical case, when the kinetic energy of the colliding atoms is equal to the smallest energy of excitation. With respect to the center of mass the atoms move in opposite direction with velocities $\tfrac{1}{2}v_0$. Thus

$$\frac{1}{2}m_H\left(\tfrac{1}{2}v_0\right)^2 + \frac{1}{2}m_H\left(\tfrac{1}{2}v_0\right)^2 = \tfrac{3}{4}E_i$$

and

$$v_0 = \sqrt{\frac{3E_i}{m_H}} \qquad (\approx 6.26 \cdot 10^4 \text{ m/s}).$$

Consider the case when $v = v_0$. The collision may be elastic or inelastic. When the collision is elastic the atoms remain in their ground states and do not emit radiation. Radiation is possible only when the collision is inelastic. Of course, only the atom excited in the collision can emit the radiation. In principle, the radiation can be emitted in any direction, but according to the text of the problem we have to consider radiation emitted in the direction of the initial velocity and in the opposite direction only. After the inelastic collision both atom are moving (in the laboratory system) with the same velocities equal to $\tfrac{1}{2}v_0$. Let $f$ denotes the frequency of radiation emitted by the hydrogen atom in the mass center (i.e. at rest). Because of the Doppler effect, in the laboratory system this frequency is observed as (*c* denotes the velocity of light):

a) $f_1 = \left(1 + \frac{\frac{1}{2}v_0}{c}\right)f$ - for radiation emitted in the direction of the initial velocity of the hydrogen atom,

b) $f_2 = \left(1 - \frac{\frac{1}{2}v_0}{c}\right)f$ - for radiation emitted in opposite direction.

The arithmetic mean value of these frequencies is equal to $f$. Thus the required ratio is

$$\frac{\Delta f}{f} = \frac{f_1 - f_2}{f} = \frac{v_0}{c} \qquad (\approx 2 \cdot 10^{-2}\%).$$

In the above solution we took into account that $v_0 << c$. Otherwise it would be necessary to use relativistic formulae for the Doppler effect. Also we neglected the recoil of atom(s) in the emission process. One should notice that for the visible radiation or radiation not too far from the visible range the recoil cannot change significantly the numerical results for the critical velocity $v_0$ and the ratio $\frac{\Delta f}{f}$. The recoil is important for high-energy quanta, but it is not this case.

The solutions were marked according to the following scheme (draft):

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|---|---|
| 1. Energy of excitation | up to 3 points |
| 2. Correct description of the physical processes | up to 4 points |
| 3. Doppler effect | up to 3 points |