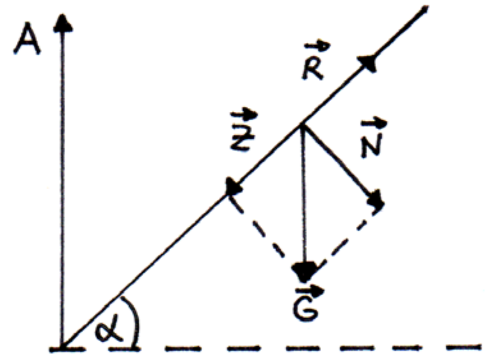
Solution of problem 1:

a) $\omega = 0$:

The forces in this case are (see figure):



$\vec{G} = \vec{Z} + \vec{N} = m \cdot \vec{g}$ (1),

$|\vec{Z}| = m \cdot g \cdot \sin\alpha = Z$ (2),

$|\vec{N}| = m \cdot g \cdot \cos\alpha = N$ (3),

$|\vec{R}| = \mu \cdot N = \mu \cdot m \cdot g \cdot \cos\alpha = R$ (4).

[$\vec{R}$: force of friction]

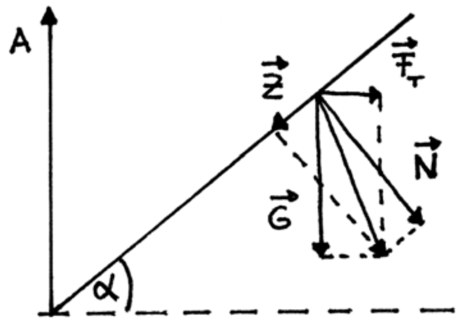
The body is at rest relative to the rod, if $Z \le R$. According to equations (2) and (4) this is equivalent to $\tan\alpha \le \tan\beta$. That means, the body is at rest relative to the rod for $\alpha \le \beta$ and the body moves along the rod for $\alpha > \beta$.

b) $\omega > 0$:

Two different situations have to be considered: 1. $\alpha > \beta$ and 2. $\alpha \le \beta$.

If the rod is moving ($\omega \ne 0$) the forces are $\vec{G} = m \cdot \vec{g}$ and $|\vec{F}_r| = m \cdot r \cdot \vec{\omega}^2$.

From the parallelogramm of forces (see figure):



$\vec{Z} + \vec{N} = \vec{G} + \vec{F}_r$ (5).

The condition of equilibrium is:

$|\vec{Z}| = \mu |\vec{N}|$ (6).

Case 1: $\vec{Z}$ is oriented downwards, i.e. $g \cdot \sin\alpha > r \cdot \omega^2 \cdot \cos\alpha$.

$$|\vec{Z}| = m \cdot g \cdot \sin\alpha - m \cdot r \cdot \omega^2 \cdot \cos\alpha \quad \text{and} \quad |\vec{N}| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

Case 2: $\vec{Z}$ is oriented upwards, i.e. $g \cdot \sin\alpha < r \cdot \omega^2 \cdot \cos\alpha$.

$$|\vec{Z}| = -m \cdot g \cdot \sin\alpha + m \cdot r \cdot \omega^2 \cdot \cos\alpha \quad \text{and} \quad |\vec{N}| = m \cdot g \cdot \cos\alpha + m \cdot r \cdot \omega^2 \cdot \sin\alpha$$

It follows from the condition of equilibrium equation (6) that

$$\pm\left(g \cdot \sin\alpha - r \cdot \omega^2 \cdot \cos\alpha\right) = \tan\beta \cdot \left(g \cdot \cos\alpha + r \cdot \omega^2 \cdot \sin\alpha\right) \quad (7).$$

Algebraic manipulation of equation (7) leads to:

$$g \cdot \sin(\alpha - \beta) = r \cdot \omega^2 \cdot \cos(\alpha - \beta) \quad (8),$$

$$g \cdot \sin(\alpha + \beta) = r \cdot \omega^2 \cdot \cos(\alpha + \beta) \quad (9).$$

That means,

$$r_{1,2} = \frac{g}{\omega^2} \cdot \tan(\alpha \mp \beta) \quad (10).$$

The body is at rest relative to the rotating rod in the case $\alpha > \beta$ if the following inequalities hold:

$$r_1 \leq r \leq r_2 \qquad \text{with } r_1, r_2 > 0 \quad (11)$$

or

$$L_1 \leq L \leq L_2 \qquad \text{with } L_1 = r_1 / \cos\alpha \text{ and } L_2 = r_2 / \cos\alpha \quad (12).$$

The body is at rest relative to the rotating rod in the case $\alpha \leq \beta$ if the following inequalities hold:

$$0 \leq r \leq r_2 \qquad \text{with } r_1 = 0 \text{ (since } r_1 < 0 \text{ is not a physical solution), } r_2 > 0 \quad (13).$$

Inequality (13) is equivalent to

$$0 \leq L \leq L_2 \qquad \text{with } L_2 = r_2 / \cos\alpha > 0 \quad (14).$$