Solution of problem 2:

a) The refractive index n is a function of the wavelength λ, i.e. $n = n(\lambda)$. According to the given formula for the focal length f (see above) which for a given f yields to an equation quadratic in n there are at most two different wavelengths (indices of refraction) for the same focal length.

b) If the focal length is the same for two different wavelengths, then the equation

$$f(\lambda_1) = f(\lambda_2) \quad or \quad f(n_1) = f(n_2) \tag{1}$$

holds. Using the given equation for the focal length it follows from equation (1):

$$\frac{n_1 r_1 r_2}{(n_1-1)\left[n_1(r_2-r_1)+d(n_1-1)\right]} = \frac{n_2 r_1 r_2}{(n_2-1)\left[n_2(r_2-r_1)+d(n_2-1)\right]}$$

Algebraic calculations lead to:

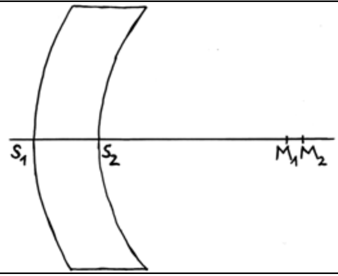
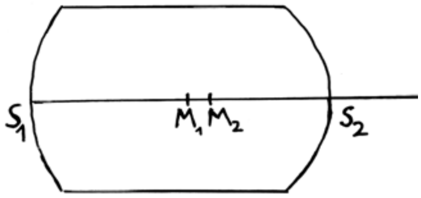
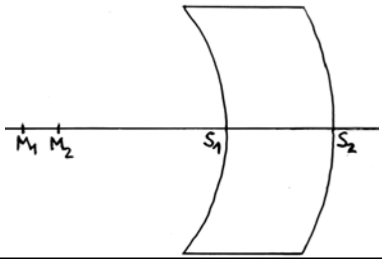
$$r_1 - r_2 = d \cdot \left(1 - \frac{1}{n_1 n_2}\right) \tag{2}.$$

If the values of the radii r_1, r_2 and the thickness satisfy this condition the focal length will be the same for two wavelengths (indices of refraction). The parameters in this equation are subject to some physical restrictions: The indices of refraction are greater than 1 and the thickness of the lens is greater than 0 m. Therefore, from equation (2) the relation

$$d > r_1 - r_2 > 0 \tag{3}$$

is obtained.

The following table shows a discussion of different cases:

r_1	r_2	***condition***	***shape of the lens***	***centre of curvature***
$r_1 > 0$	$r_2 > 0$	$0 < r_1 - r_2 < d$ or $r_2 < r_1 < d + r_2$		*M_2 is always right of M_1.* $\overline{M_1M_2} < \overline{S_1S_2}$
$r_1 > 0$	$r_2 < 0$	$r_1 + \lvert r_2 \rvert < \mathrm{d}$		*Order of points:* $S_1M_1M_2S_2$
$r_1 < 0$	$r_2 > 0$	*never fulfilled*		
$r_1 < 0$	$r_2 < 0$	$0 < \lvert r_2 \rvert - \lvert r_1 \rvert < d$ or $\lvert r_1 \rvert < \lvert r_2 \rvert < d + \lvert r_1 \rvert$		*M_2 is always right of M_1.* $\overline{M_1M_2} < \overline{S_1S_2}$

c) The radius r_1 or the radius r_2 is infinite in the case of the planconvex lens. In the following it is assumed that r_1 is infinite and r_2 is finite.

$$\lim_{r_1\to\infty} f = \lim_{r_1\to\infty} \frac{n\,r_2}{(n-1)\left[n\left(\frac{r_2}{r_1}-1\right)+(n-1)\frac{d}{r_1}\right]} = \frac{r_2}{1-n} \qquad (4)$$

Equation (4) means, that for each wavelength (refractive index) there exists a different value of the focal length.

d) From the given formula for the focal length (see problem formulation) one obtains the following quadratic equation in n:

$$A\cdot n^2 + B\cdot n + C = 0 \qquad (5)$$

with $A = (r_2 - r_1 + d)\cdot f$, $B = -\left[f\cdot(r_2 - r_1) + 2\cdot f\cdot d + r_1\cdot r_2\right]$ and $C = f\cdot d$.

Solutions of equation (5) are:

$$n_{1,2} = -\frac{B}{2 \cdot A} \pm \sqrt{\frac{B^2}{4 \cdot A^2} - \frac{C}{A}} \quad (6).$$

Equation (5) has only one physical correct solution, if...

I) A = 0 (i.e., the coefficient of n^2 in equation (5) vanishes)

In this case the following relationships exists:

$$r_1 - r_2 = d \quad (7),$$

$$n = \frac{f \cdot d}{f \cdot d + r_1 \cdot r_2} > 1 \quad (8).$$

II) B = 0 (i.e. the coefficient of *n* in equation (5) vanishes)

In this case the equation has a positive and a negative solution. Only the positve solution makes sense from the physical point of view. It is:

$$f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2 = 0 \quad (9),$$

$$n^2 = -\frac{C}{A} = -\frac{d}{(r_2 - r_1 + d)} > 1 \quad (10),$$

III) $B^2 = 4$ AC

In this case two identical real solutions exist. It is:

$$\left[f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2\right]^2 = 4 \cdot (r_2 - r_1 + d) \cdot f^2 \cdot d \quad (11),$$

$$n = -\frac{B}{2 \cdot A} = \frac{f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2}{2 \cdot f\,(r_2 - r_1 + d)} > 1 \quad (12).$$