Solution of problem 3:

a) The kinetic energy of the ion after acceleration by a voltage U is:

$$\tfrac{1}{2} mv^2 = eU \qquad (1).$$

From equation (1) the velocity of the ions is calculated:

$$v = \sqrt{\frac{2 \cdot e \cdot U}{m}} \qquad (2).$$

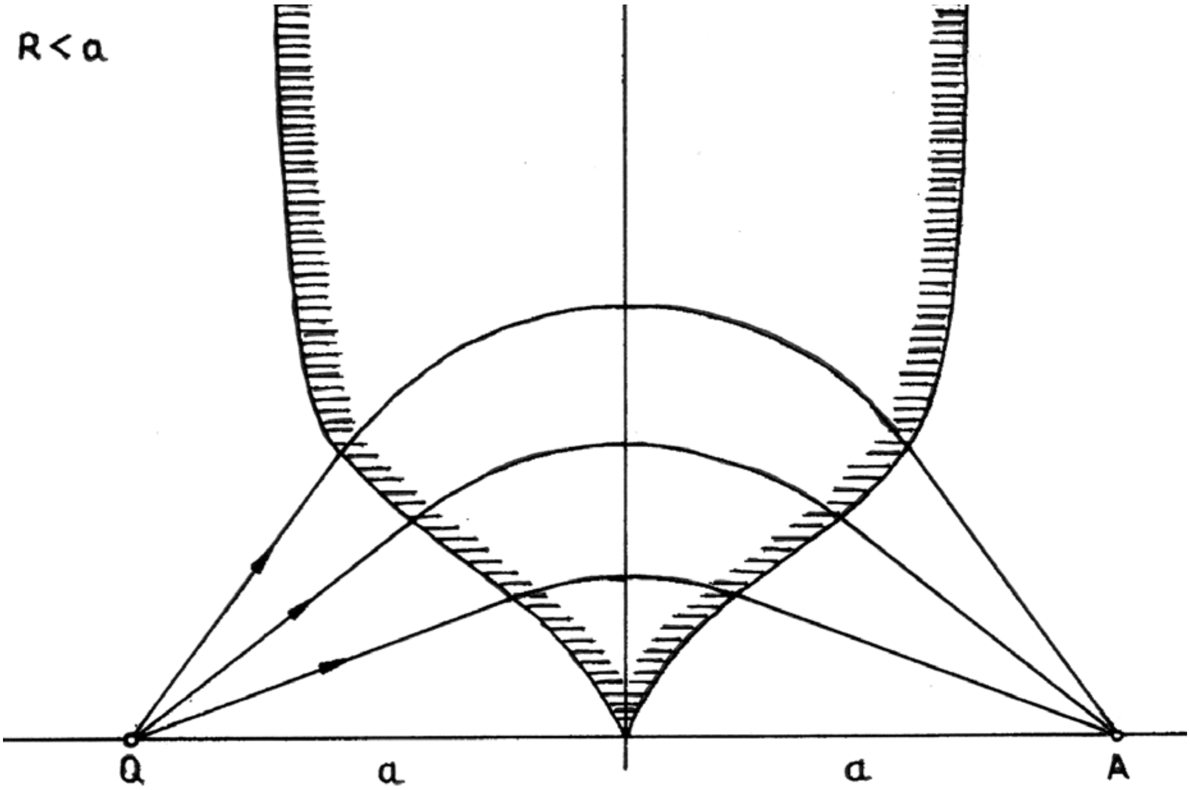
On a moving ion (charge *e* and velocity *v*) in a homogenous magnetic field *B* acts a Lorentz force *F*. Under the given conditions the velocity is always perpendicular to the magnetic field. Therefore, the paths of the ions are circular with Radius *R*. Lorentz force and centrifugal force are of the same amount:

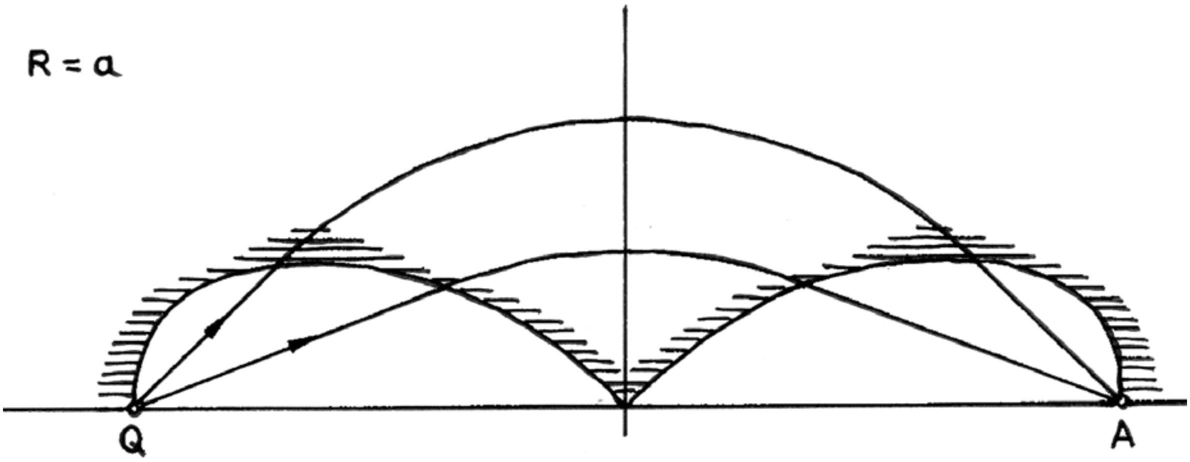
$$e \cdot v \cdot B = \frac{m \cdot v^2}{R} \qquad (3).$$

From equation (3) the radius of the ion path is calculated:

$$R = \frac{1}{B}\sqrt{\frac{2 \cdot m \cdot U}{e}} \qquad (4).$$

b) All ions of mass *m* travel on circular paths of radius $R = v{\cdot}m / e{\cdot}B$ inside the magnetic field. Leaving the magnetic field they fly in a straight line along the last tangent. The centres of curvature of the ion paths lie on the middle perpendicular on $\overline{QA}$ since the magnetic field is assumed to be symmetric to the middle perpendicular on $\overline{QA}$. The paths of the focussed ions are above $\overline{QA}$ due to the direction of the magnetic field.


R < a
Q
a
a
A


R = a
Q
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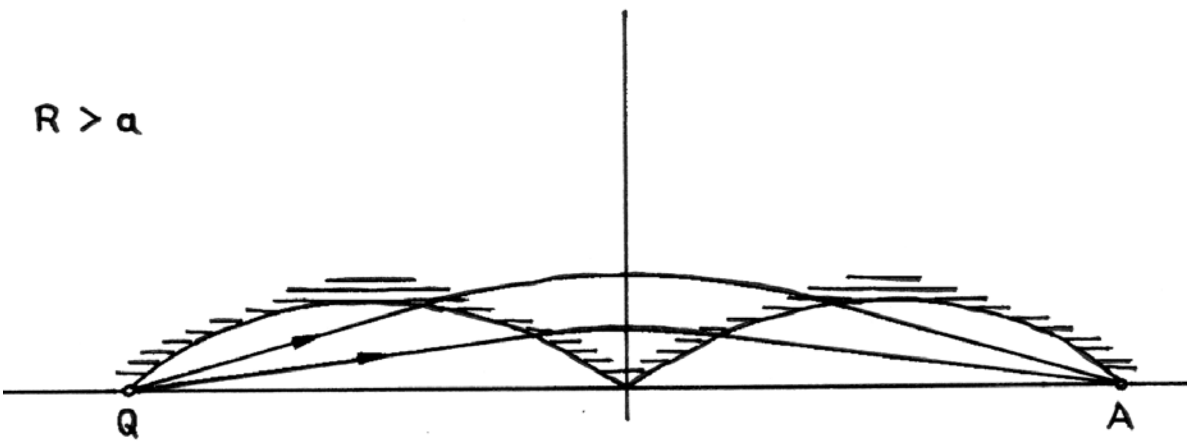
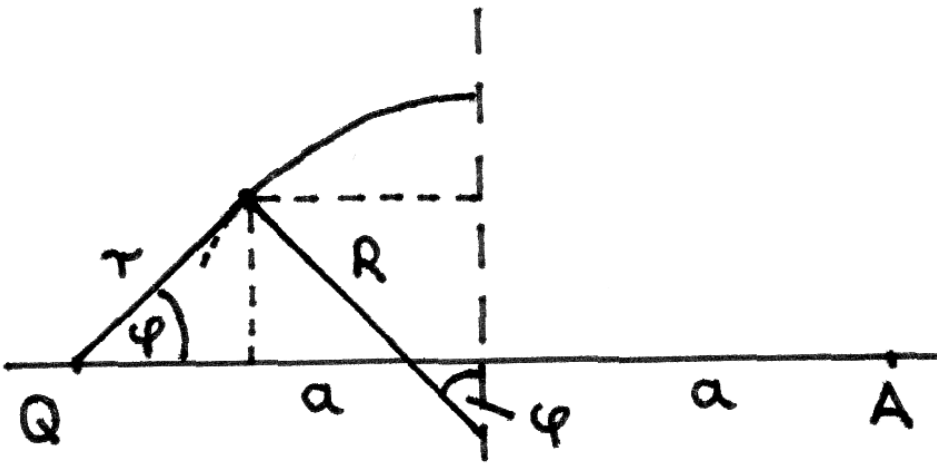

R > a
Q
A

c) The construction method of the boundaries of the magnetic fields is based on the considerations in part b:

- Sketch circles of radius R and different centres of curvature on the middle perpendicular on $\overline{\mathrm{QA}}$.
- Sketch tangents on the circle with either point Q or point A on these straight lines.
- The points of tangency make up the boundaries of the magnetic field. If $R > a$ then not all ions will reach point A. Ions starting at an angle steeper than the tangent at Q, do not arrive in A. The figure on the last page shows the boundaries of the magnetic field for the three cases $R < a$, $R = a$ and $R > a$.

d) It is convenient to deduce a general equation for the boundaries of the magnetic field in polar coordinates (r, φ) instead of using cartesian coordinates (x, y).



The following relation is obtained from the figure:

$$r \cdot \cos\varphi + R \sin\varphi = a \qquad (7).$$

The boundaries of the magnetic field are given by:

$$r = \frac{a}{\cos\varphi}\left(1 - \frac{R}{a}\sin\varphi\right) \qquad (8).$$