Solution

a) The block moves along a horizontal circle of radius $R\sin\alpha$. The net force acting on the block is pointed to the centre of this circle (*Fig. 7*). The vector sum of the normal force exerted by the wall *N*, the frictional force *S* and the weight *mg* is equal to the resultant: $m\omega^2 R\sin\alpha$.

The connections between the horizontal and vertical components:

$$m\omega^2 R\sin\alpha = N\sin\alpha - S\cos\alpha\,,$$

$$mg = N\cos\alpha + S\sin\alpha\,.$$

The solution of the system of equations:

$$S = mg\sin\alpha\left(1-\frac{\omega^2 R\cos\alpha}{g}\right),$$

$$N = mg\left(\cos\alpha+\frac{\omega^2 R\sin^2\alpha}{g}\right).$$

The block does not slip down if

$$\mu_a \geq \frac{S}{N} = \sin\alpha\cdot\frac{1-\dfrac{\omega^2 R\cos\alpha}{g}}{\cos\alpha+\dfrac{\omega^2 R\sin^2\alpha}{g}} = \frac{3\sqrt{3}}{23} = \mathbf{0.2259}\,.$$

In this case there must be at least this friction to prevent slipping, i.e. sliding down.

b) If on the other hand $\dfrac{\omega^2 R\cos\alpha}{g} > 1$ some friction is necessary to prevent the block to slip upwards. $m\omega^2 R\sin\alpha$ must be equal to the resultant of forces *S*, *N* and *mg*. Condition for the minimal coefficient of friction is (*Fig. 8*):

$$\mu_b \geq \frac{S}{N} = \sin\alpha\cdot\frac{\dfrac{\omega^2 R\cos\alpha}{g}-1}{\cos\alpha+\dfrac{\omega^2 R\sin^2\alpha}{g}} =$$

$$= \frac{3\sqrt{3}}{29} = \mathbf{0.1792}\,.$$

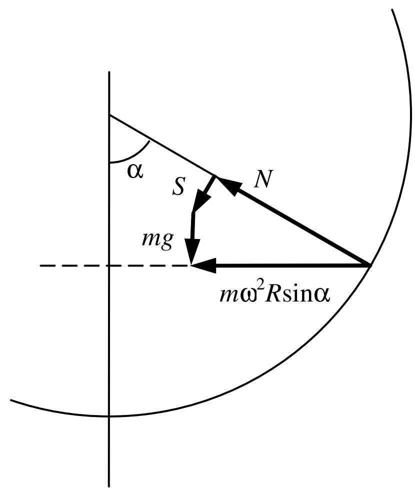

Figure 8

c) We have to investigate μ_a and μ_b as functions of α and ω in the cases a) and b) (see *Fig. 9/a* and *9/b*):

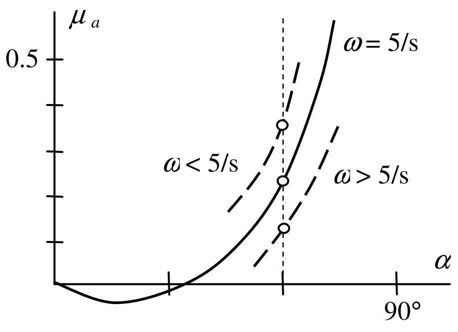

Figure 9/a

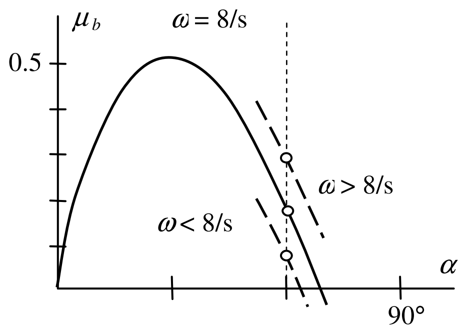

Figure 9/b

In case a): if the block slips upwards, it comes back; if it slips down it does not return. If ω increases, the block remains in equilibrium, if ω decreases it slips downwards.

In case b): if the block slips upwards it stays there; if the block slips downwards it returns. If ω increases the block climbs upwards, if ω decreases the block remains in equilibrium.