

Solution: The thin layer reflects the monochromatic light of the wavelength λ in the best way, if the following equation holds true

$$2nd \cos \beta = (2k + 1) \frac{\lambda}{2}, \quad k = 0, 1, 2, \dots, \quad (1)$$

where k denotes an integer and β is the angle of refraction satisfying

$$\frac{\sin \alpha}{\sin \beta} = n.$$

Hence,

$$\cos \beta = \sqrt{1 - \sin^2 \beta} = \frac{1}{n} \sqrt{n^2 - \sin^2 \alpha}.$$

Substituting to (1) we obtain

$$2d \sqrt{n^2 - \sin^2 \alpha} = (2k + 1) \frac{\lambda}{2}. \quad (2)$$

If the white light falls on a layer, the colors of wavelengths obeying (2) are reinforced in the reflected light. If the wavelength of the reflected light is λ_0 , the thickness of the layer satisfies for the k th order interference

$$d_k = \frac{(2k + 1)\lambda_0}{4\sqrt{n^2 - \sin^2 \alpha}} = (2k + 1)d_0.$$

For given values and $k = 0$ we obtain $d_0 = 1.01 \cdot 10^{-7}$ m.

a) The mass of the soap film is $m_k = \rho_k b h d_k$. Substituting the given values, we get $m_0 = 6.06 \cdot 10^{-2}$ mg, $m_1 = 18.2 \cdot 10^{-2}$ mg, $m_2 = 30.3 \cdot 10^{-8}$ mg, etc. The mass of the thinnest film thus cannot be determined by given laboratory scales.

b) If the light falls at the angle of 30° then the film seen from the perpendicular direction cannot be colored. It would appear dark.