Solution: a) If a uniform magnetic field is perpendicular to the initial direction of motion of an electron beam, the electrons will be deflected by a force that is always perpendicular to their velocity and to the magnetic field. Consequently, the beam will be deflected into a circular trajectory. The origin of the centripetal force is the Lorentz force, so

$$Bev = \frac{m_e v^2}{r} \,. \tag{3}$$

Geometrical considerations yield that the radius of the trajectory obeys (cf. Fig. 3).

$$r = \frac{d}{2\sin\alpha} \,. \tag{4}$$

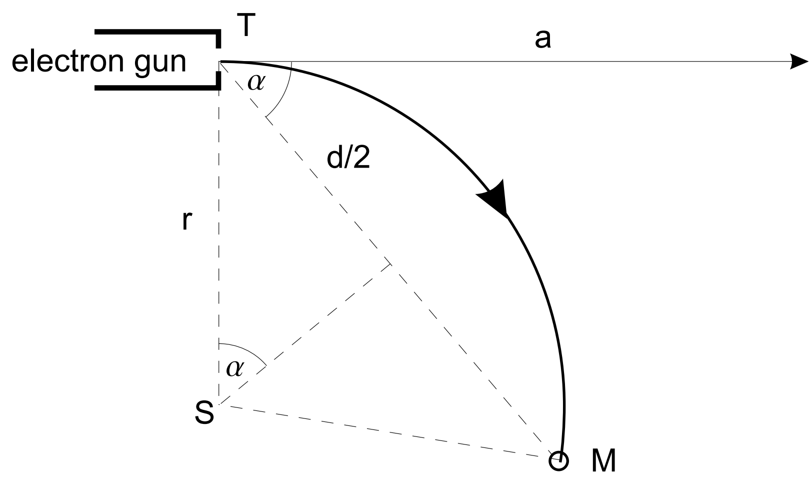

Figure 3:

The velocity of electrons can be determined from the relation between the kinetic energy of an electron and the work done on this electron by the electric field of the voltage U inside the gun,

$$\frac{1}{2}m_e v^2 = eU\,. \tag{5}$$

Using (3), (4) and (5) one obtains

$$B = m_e\sqrt{\frac{2eU}{m_e}}\frac{2\sin\alpha}{ed} = 2\sqrt{\frac{2Um_e}{e}}\frac{\sin\alpha}{d}\,.$$

Substituting the given values we have $B = 3.70 \cdot 10^{-3}$ T.

b) If a uniform magnetic field is neither perpendicular nor parallel to the initial direction of motion of an electron beam, the electrons will be deflected into a helical trajectory. Namely, the motion of electrons will be composed of an uniform motion on a circle in the plane perpendicular to the magnetic field and of an uniform rectilinear motion in the direction of the magnetic field. The component $\vec{v}_1$ of the initial velocity $\vec{v}$, which is perpendicular to the magnetic field (see Fig. 4), will manifest itself at the Lorentz force and during the motion will rotate uniformly around the line parallel to the magnetic field. The component $\vec{v}_2$ parallel to the magnetic field will remain

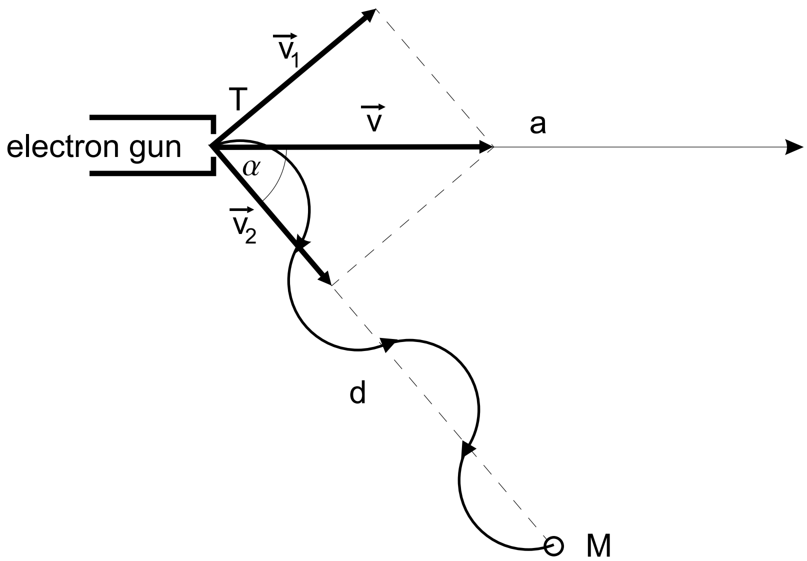

Figure 4:

constant during the motion, it will be the velocity of the uniform rectilinear motion. Magnitudes of the components of the velocity can be expressed as

$$v_1 = v \sin\alpha \quad v_2 = v \cos\alpha \,.$$

Denoting by N the number of screws of the helix we can write for the time of motion of the electron

$$t = \frac{d}{v_2} = \frac{d}{v\cos\alpha} = \frac{2\pi r N}{v_1} = \frac{2\pi r N}{v \sin\alpha} \,.$$

Hence we can calculate the radius of the circular trajectory

$$r = \frac{d \sin\alpha}{2\pi N \cos\alpha} \,.$$

However, the Lorentz force must be equated to the centripetal force

$$Bev\sin\alpha = \frac{m_e v^2 \sin^2\alpha}{r} = \frac{m_e v^2 \sin^2\alpha}{\frac{d\sin\alpha}{2\pi N\cos\alpha}} \,. \tag{6}$$

Consequently,

$$B = \frac{m_e v^2 \sin^2\alpha \, 2\pi N \cos\alpha}{d \sin\alpha \, ev \sin\alpha} = \frac{2\pi N m_e v \cos\alpha}{de} \,.$$

The magnitude of velocity v again satisfies (5), so

$$v = \sqrt{\frac{2Ue}{m_e}} \,.$$

Substituting into (6) one obtains

$$B = \frac{2\pi N \cos\alpha}{d} \sqrt{\frac{2Um_e}{e}} \,.$$

Numerically we get $B = N \cdot 6.70 \cdot 10^{-3}$ T . If $B < 0.030$ T should hold true, we have four possibilities ($N \leq 4$). Namely,

$$B_1 = 6.70 \cdot 10^{-3} \text{ T},$$
$$B_2 = 13.4 \cdot 10^{-3} \text{ T},$$
$$B_3 = 20.1 \cdot 10^{-3} \text{ T},$$
$$B_4 = 26.8 \cdot 10^{-3} \text{ T}.$$