Problem 3.

1) The beam divergence angle $\delta\varphi$ caused by diffraction defines the accuracy of the telescope optical axis installation:

$$\delta\varphi \approx \lambda/D \approx 2.6\cdot10^{-7}\text{ rad.} \approx 0.05''.$$

2) The part K_1 of the light energy of a laser, directed to a reflector, may be found by the ratio of the area of S_1 reflector ($S_1 = \pi d^2/4$) versus the area S_2 of the light spot on the Moon ($S_2 = \pi r^2$, where $r = L\,\delta\varphi \approx L\lambda/D$, L – the distance from the Earth to the Moon)

$$K_1 = \frac{S_1}{S_2} = \frac{d^2}{(2r)^2} = \frac{d^2D^2}{4\lambda^2L^2}$$

The reflected light beam diverges as well and forms a light spot with the radius R on the Earth's surface:

$$R = \lambda L/d, \quad \text{as} \quad r << R$$

That's why the part K_2 of the reflected energy, which got into the telescope, makes

$$K_2 = \frac{D^2}{(2R)^2} = \frac{D^2d^2}{4\lambda^2L^2}$$

The part K_0 of the laser energy, that got into the telescope after having been reflected by the reflector on the Moon, equals

$$K_0 = K_1K_2 = \left(\frac{dD}{2\lambda L}\right)^4 \approx 10^{-12}$$

3) The pupil of a naked eye receives as less a part of the light flux compared to a telescope, as the area of the pupil S_e is less than the area of the telescope mirror S_t:

$$K_e = K_0\frac{S_e}{S_t} = K_0\frac{d_e^2}{D^2} \approx 3.7\cdot10^{-18}.$$

So the number of photons N getting into the pupil of the eye is equal

$$N = \frac{E}{h\nu}K_e = 12.$$

Since $N<n$, one can not perceive the reflected pulse with a naked eye.

4) In the absence of a reflector α =10% of the laser energy, that got onto the Moon, are dispersed by the lunar surface within a solid angle $\Omega_1 = 2\pi$ steradian.

The solid angle in which one can see the telescope mirror from the Moon, constitutes

$$\Omega_2 = S_t/L^2 = \pi D^2/4L^2$$

That is why the part K of the energy gets into the telescope and it is equal

$$K = \alpha\frac{\Omega_2}{\Omega_1} = \alpha\frac{D^2}{8L^2} \approx 0.5\cdot10^{-18}$$

Thus, the gain β, which is obtained through the use of the reflector is equal

$$\beta = K_0/K \approx 2 \cdot 10^6$$

Note. The result obtained is only evaluative as the light flux is unevenly distributed inside the angle of diffraction.