Solution of the Theoretical Problem 1

Up to the moment when the piston leaves the container, the system can be considered as a closed one. It follows from the laws of the conservation of the momentum and the energy:

$$(M+nM_0)v_1 - mu = 0 \tag{1}$$

$$\frac{(M+nM_0)v_1^2}{2} + \frac{mu^2}{2} = \Delta U\ , \tag{2}$$

where v_1 – velocity of the container when the piston leaves it, u – velocity of the piston in the same moment, ΔU – the change of the internal energy of the gas. The gas is perfect and monoatomic, therefore

$$\Delta U = \frac{3}{2}nR\Delta T = \frac{3}{2}nR(T - T_f)\,; \tag{3}$$

T_f - the temperature of the gas in the moment when the piston leaves the container. This temperature can be determined by the law of the adiabatic process:

$$pV^{\gamma} = const.$$

Using the perfect gas equation $pV = nRT$, one obtains

$$TV^{\gamma-1} = const.,\qquad TV^{\gamma-1} = T_f V_f^{\gamma-1}$$

Using the relation $V_f = 2V$, and the fact that the adiabatic coefficient for one-atomic gas is $\gamma = \frac{c_p}{c_v} = \frac{\frac{5}{2}R}{\frac{3}{2}R} = \frac{5}{3}$, the result for final temperature is:

$$T_f = T\left(\frac{V}{V_f}\right)^{\gamma-1} = \frac{T}{2^{2/3}} = T2^{-2/3} \tag{4}$$

Solving the equations (1) – (4) we obtain

$$v_1 = \sqrt{3(1-2^{-2/3})\frac{mnRT}{(nM_0+M)(m+nM_0+M)}} \tag{5}$$

If the gas mass nM_0 is much smaller than the masses of the container M and the piston m, then the equation (5) is simplified to:

$$v_1 = \sqrt{3(1-2^{-2/3})\frac{mnRT}{M(m+M)}} \tag{5'}$$

When the piston leaves the container, the velocity of the container additionally increases to value v_2 due to the hits of the atoms in the bottom of the container. Each atom gives the container momentum:

$$p = 2m_A\Delta\overline{v_x}\ ,$$

where m_A – mass of the atom; $m_A = \frac{M_0}{N_A}$, and $\overline{v_x}$ can be obtained by the averaged quadratic velocity of the atoms $\overline{v^2}$ as follows:

$\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = \overline{v^2}$, and $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$, therefore $\overline{v_x} = \sqrt{\frac{\overline{v^2}}{3}}$. It appears that due to the elastic impact of one atom the container receives averaged momentum

$$p = 2\frac{M_0}{N_A}\sqrt{\frac{\overline{v^2}}{3}}$$

All calculations are done assuming that the thermal velocities of the atoms are much larger than the velocity of the container and that the movement is described using system connected with the container.

Have in mind that only half of the atoms hit the bottom of the container, the total momentum received by the container is

$$p_t = \frac{1}{2}nN_A p = nM_0\sqrt{\frac{\overline{v^2}}{3}} \qquad (6)$$

and additional increase of the velocity of the container is

$$v_2 = \frac{p_t}{M} = n\frac{M_0}{M}\sqrt{\frac{\overline{v^2}}{3}}\ . \qquad (7)$$

Using the formula for the averaged quadratic velocity

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3RT_f}{M_0}}$$

as well eq. (4) for the temperature T_f, the final result for v_2 is

$$v_2 = 2^{-1/3}\frac{n\sqrt{M_0RT}}{M}\ . \qquad (8)$$

Therefore the final velocity of the container is

$$v = v_1 + v_2 = \sqrt{3(1-2^{-2/3})\frac{mnRT}{(nM_0+M)(m+nM_0+M)}} + 2^{-1/3}\frac{n\sqrt{M_0RT}}{M} \approx$$

$$\approx \sqrt{3(1-2^{-2/3})\frac{mnRT}{M(m+M)}} + 2^{-1/3}\frac{n\sqrt{M_0RT}}{M}\ . \qquad (9)$$