Optics – Problem III (7points)

Problem III - Solution

a. The ray with the wavelength λ_0 pass trough the prisms system without refraction on AC face at any angle of incidence if :

$n_1(\lambda_0) = n_2(\lambda_0)$

Because the dependence of refraction indexes of prisms on wavelength has the form :

$$n_1(\lambda) = a_1 + \frac{b_1}{\lambda^2} \quad (3.1)$$

$$n_2(\lambda) = a_2 + \frac{b_2}{\lambda^2} \quad (3.2)$$

The relation (3.1) becomes:

$$a_1 + \frac{b_1}{\lambda_0^2} = a_2 + \frac{b_2}{\lambda_0^2} \quad (3.3)$$

The wavelength λ_0 has correspondingly the form:

$$\lambda_0 = \sqrt{\frac{b_1 - b_2}{a_2 - a_1}} \quad (3.4)$$

Substituting the furnished numerical values

$$\lambda_0 = 500 nm \quad (3.5)$$

The corresponding common value of indexes of refraction of prisms for the radiation with the wavelength λ_0 is:

$$n_1(\lambda_0) = n_2(\lambda_0) = 1,5 \quad (3.6)$$

The relations (3.6) and (3.7) represent the answers of question **a.**

b. For the rays with different wavelength (λ_{red}, λ_0, λ_{violet}) having the same incidence angle on first prism, the paths are illustrated in the figure 1.1.

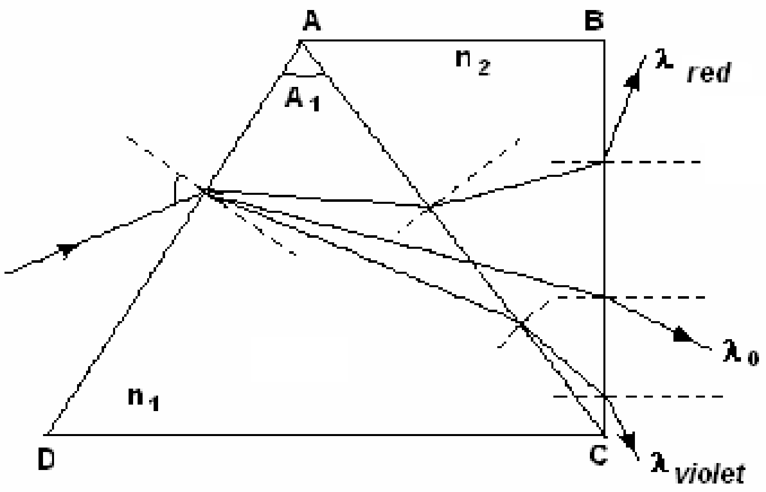

Figure 3.1

The draw illustrated in the figure 1.1 represents the answer of question **b.**

c. In the figure 1.2 is presented the path of ray with wavelength λ_0 at minimum deviation (the angle between the direction of incidence of ray and the direction of emerging ray is minimal).

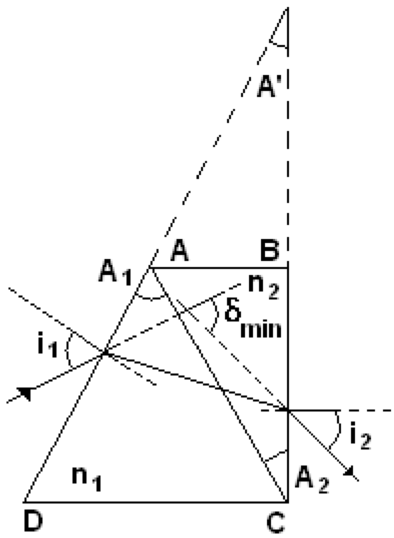

Figure 3.2

In this situation

$$n_1(\lambda_0) = n_2(\lambda_0) = \frac{\sin\frac{\delta_{\min} + A'}{2}}{\sin\frac{A'}{2}} \qquad (3.7)$$

where

$m(\hat{A}') = 30°$,

as in the figure 1.1

Substituting in (3.8) the values of refraction indexes the result is

$$\sin\frac{\delta_{\min} + A'}{2} = \frac{3}{2}\cdot\sin\frac{A'}{2} \qquad (3.8)$$

or

$$\delta_{\min} = 2\arcsin\left(\frac{3}{2}\cdot\sin\frac{A'}{2}\right) - \frac{A'}{2} \qquad (3.9)$$

Numerically

$$\delta_{\min} \cong 30{,}7° \qquad (3.10)$$

The relation (3.11) represents the answer of question **c**.

d. Using the figure 1.3 the refraction law on the AD face is

$$\sin i_1 = n_1 \cdot \sin r_1 \qquad (3.11)$$

The refraction law on the AC face is

$$n_1 \cdot \sin r_1' = n_2 \cdot \sin r_2 \qquad (3.12)$$

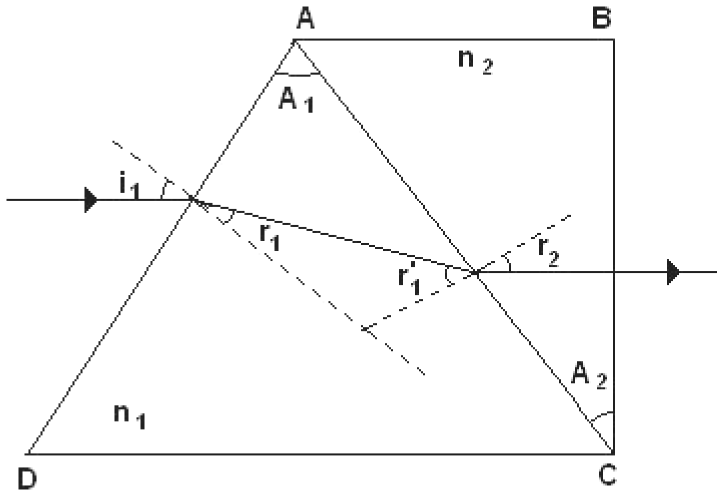

Figure 3.3

As it can be seen in the figure 1.3

$$r_2 = A_2 \tag{3.13}$$

and

$$i_1 = 30° \tag{3.14}$$

Also,

$$r_1 + r_1' = A_1 \tag{3.15}$$

Substituting (3.16) and (3.14) in (3.13) it results

$$n_1 \cdot \sin\left(A_1 - r_1\right) = n_2 \cdot \sin A_2 \tag{3.16}$$

or

$$n_1 \cdot \left(\sin A_1 \cdot \cos r_1 - \sin r_1 \cdot \cos A_1\right) = n_2 \cdot \sin A_2 \tag{3.17}$$

Because of (3.12) and (3.15) it results that

$$\sin r_1 = \frac{1}{2n_1} \tag{3.18}$$

and

$$\cos r_1 = \frac{1}{2n_1}\sqrt{4n_1^{\ 2} - 1} \tag{3.19}$$

Putting together the last three relations it results

$$\sqrt{4n_1^{\ 2} - 1} = \frac{2n_2 \cdot \sin A_2 + \cos A_1}{\sin A_1} \tag{3.20}$$

Because

$\hat{A}_1 = 60°$

and

$\hat{A}_2 = 30°$

relation (3.21) can be written as

$$\sqrt{4n_1^{\ 2} - 1} = \frac{2n_2 + 1}{\sqrt{3}} \tag{3.21}$$

or

$3 \cdot n_1{}^2 = 1 + n_2 + n_2{}^2$ (3.22)

Considering the relations (3.1), (3.2) and (3.23) and operating all calculus it results:

$\lambda^4 \cdot \left(3 a_1{}^2 - a_2{}^2 - a_2 - 1\right) + \left(6 a_1 b_1 - b_2 - 2 a_2 b_2\right) \cdot \lambda^2 + 3 b_1{}^2 - b_2{}^2 = 0$ (3.23)

Solving the equation (3.24) one determine the wavelength λ of the ray that enter the prisms system having the direction parallel with DC and emerges the prism system having the direction again parallel with DC. That is

$\lambda = 1194 nm$ (3.24)

or

$\lambda \cong 1{,}2 \mu m$ (3.25)

The relation (3.26) represents the answer of question **d**.

Professor Delia DAVIDESCU, National Department of Evaluation and Examination–Ministry of Education and Research- Bucharest, Romania
Professor Adrian S.DAFINEI,PhD, Faculty of Physics – University of Bucharest, Romania