

## 2 Solutions

### 2.1 Theoretical competition

#### Problem 1

a) Let the electrical signals supplied to rods 1 and 2 be  $E_1 = E_0 \cos \omega t$  and  $E_2 = E_0 \cos(\omega t + \delta)$ , respectively. The condition for a maximum signal in direction  $\vartheta_A$  (Fig. 4) is:

$$\frac{2\pi a}{\lambda} \sin \vartheta_A - \delta = 2\pi N$$

and the condition for a minimum signal in direction  $\vartheta_B$ :

$$\frac{2\pi a}{\lambda} \sin \vartheta_B - \delta = 2\pi N' + \pi \quad (2p.)$$

where  $N$  and  $N'$  are arbitrary integers. In addition,  $\vartheta_A - \vartheta_B = \varphi$ , where

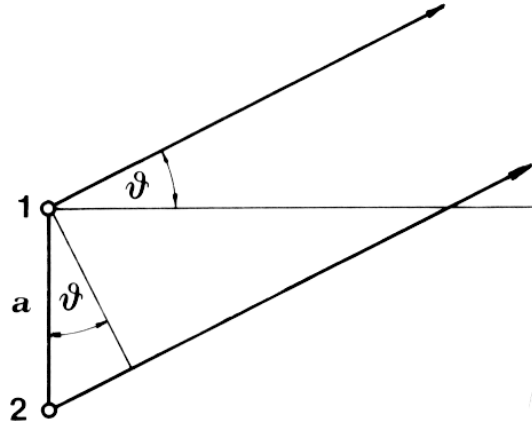


Fig. 4

$\varphi$  is given. The problem can now be formulated as follows: Find the parameters  $a$ ,  $\vartheta_A$ ,  $\vartheta_B$ ,  $\delta$ ,  $N$ , and  $N'$  satisfying the above equations such, that  $a$  is minimum.

We first eliminate  $\delta$  by subtracting the second equation from the first one:

$$a \sin \vartheta_A - a \sin \vartheta_B = \lambda(N - N' - \frac{1}{2}).$$

Using the sine addition theorem and the relation  $\vartheta_B = \vartheta_A - \varphi$ :

$$2a \cos(\vartheta_A - \frac{1}{2}\varphi) \sin \frac{1}{2}\varphi = \lambda(N - N' - \frac{1}{2})$$

or

$$a = \frac{\lambda(N - N' - \frac{1}{2})}{2 \cos(\vartheta_A - \frac{1}{2}\varphi) \sin \frac{1}{2}\varphi}.$$

The minimum of  $a$  is obtained for the greatest possible value of the denominator, i. e.:

$$\cos(\vartheta_A - \frac{1}{2}\varphi) = 1, \quad \vartheta_A = \frac{1}{2}\varphi,$$

and the minimum value of the numerator, i. e.:

$$N - N' = 1.$$

The solution is therefore:

$$a = \frac{\lambda}{4 \sin \frac{1}{2}\varphi}, \quad \vartheta_A = \frac{1}{2}\varphi, \quad \vartheta_B = -\frac{1}{2}\varphi \quad \text{and} \quad \delta = \frac{1}{2}\pi - 2\pi N. \quad (6p.)$$

( $N = 0$  can be assumed throughout without losing any physically relevant solution.)

b) The wavelength  $\lambda = c/\nu = 11.1$  m, and the angle between directions A and B,  $\varphi = 157^\circ - 72^\circ = 85^\circ$ . The minimum distance between the rods is  $a = 4.1$  m, while the direction of the symmetry line of the rods is  $72^\circ + 42.5^\circ = 114.5^\circ$  measured from the north. (2 p.)