Q2

International Physics Olympiad 1956

2. Early this century a model of the earth was proposed in which it was assumed to be a sphere of radius R consisting of a homogeneous isotropic solid mantle down to radius R_c. The core region within radius R_c contained a liquid. Figure 2.1

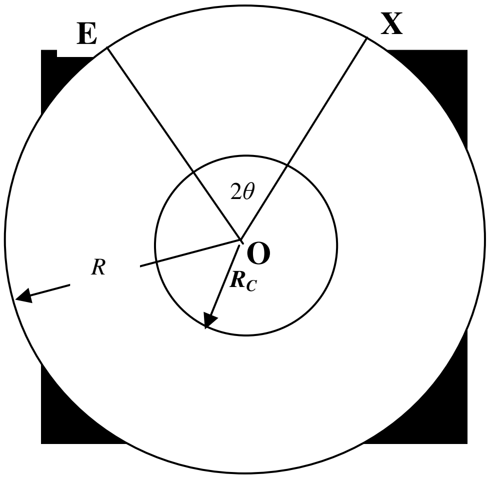

Figure 2.1

The velocities of longitudinal and transverse seismic waves P and S waves respectively, are constant, V_P, and V_S within the mantle. In the core, longitudinal waves have a constant velocity V_{CP}, < V_P, and transverse waves are not propagated.

An earthquake at E on the surface of the Earth produces seismic waves that travel through the Earth and are observed by a surface observer who can set up his seismometer at any point X on the Earth's surface. The angular separation between E and X, 2θ given by

$$2\theta = \text{Angle } EOX$$

where O is the centre of the Earth.

(i) Show that the seismic waves that travel through the mantle in a straight line will arrive at X at a time t (the travel time after the earthquake), is given by

$$t = \frac{2R\sin\theta}{v}, \qquad \text{for } \theta > \arccos\left[\frac{R_c}{R}\right],$$

where $v = v_P$ for the P waves and $v = v_S$ for the S waves.

(ii) For some of the positions of X such that the seismic P waves arrive at the observer after two refractions at the mantle-core interface. Draw the path of such a seismic P wave. Obtain a relation between θ and i, the angle of incidence of the seismic P wave at the mantle-core interface, for P waves.

(iii) Using the data

$$
\begin{aligned}
R &= 6370 \text{ km} \\
R_C &= 3470 \text{ km} \\
v_{CP} &= 10.85 \text{ km s}^{-1} \\
v_S &= 6.31 \text{ km s}^{-1} \\
v_{CP} &= 9.02 \text{ km s}^{-1}
\end{aligned}
$$

and the result obtained in (ii),draw a graph of θ against i. Comment on the physical consequences of the form of this graph for observers stationed at different points on the Earth's surface. Sketch the variation of the travel time taken by the P and S waves as a function of θ for $0 \leq \theta \leq 90$ degrees.

(iv) After an earthquake an observer measures the time delay between the arrival of the S wave, following the P wave, as 2 minutes 11 seconds. Deduce the angular separation of the earthquake from the observer using the data given in Section (iii).

(v) The observer in the previous measurement notices that some time after the arrival of the P and S waves there are two further recordings on the seismometer separated by a time interval of 6 minutes 37 seconds. Explain this result and verify that it is indeed associated with the angular separation determined in the previous section.