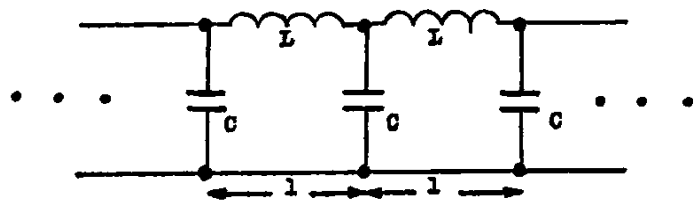
Problem 3: Infinite LC-grid

When sine waves propagate in an infinite LC-grid (see the figure below) the phase of the ac-voltage across two successive capacitors differs by Φ.



a) Determine how Φ depends on ω, L and C (ω is the angular frequency of the sine wave).

b) Determine the velocity of propagation of the waves if the length of each unit is ℓ .

c) State under what conditions the propagation velocity of the waves is almost independent of ω. Determine the velocity in this case.

d) Suggest a simple mechanical model which is an analogue to the above circuit and derive equations which establish the validity of your model.

Formulae:

$$\cos\alpha - \cos\beta = -2 \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2 \cdot \cos\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha-\beta}{2}\right)$$