Solution

2.1

conservation of energy: $M \cdot g \cdot s = \frac{1}{2} \cdot I_A \cdot \omega^2$ .... (1)

where ω is the angular speed of the wheel and I_A is the moment of inertia about the axis through A.

Note: If we would take the moment of inertia about S instead of A we would have

$$M \cdot g \cdot s = \frac{1}{2} \cdot I_S \cdot \omega^2 + \frac{1}{2} \cdot m \cdot v^2$$

where v is the speed of the centre of mass along the vertical.
This equation is the same as the above one in meanings since

$I_A = I_S + M \cdot r^2$ and $I_S = M \cdot R^2$

From (1) we get $\omega = \sqrt{\frac{2 \cdot M \cdot g \cdot s}{I_A}}$

substitute $I_A = \frac{1}{2} \cdot M \cdot r^2 + M \cdot R^2$

$$\omega = \sqrt{\frac{2 \cdot g \cdot s}{r^2 + \frac{R^2}{2}}}$$

Putting in numbers we get $\boxed{\omega = \sqrt{\frac{2 \cdot 9{,}81 \cdot 0{,}50}{9 \cdot 10^{-6} + \frac{1}{2} \cdot 36 \cdot 10^{-4}}} \approx 72{,}4 \frac{\text{rad}}{\text{s}}}$

2.2

Kinetic energy of linear motion of the centre of mass of the wheel is

$$E_T = \frac{1}{2} \cdot M \cdot v^2 = \frac{1}{2} \cdot M \cdot \omega^2 \cdot r^2 = \frac{1}{2} \cdot 0{,}40 \cdot 72{,}4^2 \cdot 9 \cdot 10^{-6} = 9{,}76 \cdot 10^{-3} \text{ J}$$

Potential energy of the wheel

$$E_P = M \cdot g \cdot s = 0{,}40 \cdot 9{,}81 \cdot 0{,}50 = 1{,}962 \text{ J}$$

Rotational kinetic energy of the wheel

$$E_R = \frac{1}{2} \cdot I_S \cdot \omega^2 = \frac{1}{2} \cdot 0{,}40 \cdot 1{,}81 \cdot 10^{-3} \cdot 72{,}4^2 = 1{,}899 \text{ J}$$

$$\boxed{\frac{E_T}{E_R} = \frac{9{,}76 \cdot 10^{-3}}{1{,}899} = 5{,}13 \cdot 10^{-3}}$$

2.3

Let $\frac{T}{2}$ be the tension in each string.

Torque τ which causes the rotation is given by $\tau = M \cdot g \cdot r = I_A \cdot \alpha$

where α is the angular acceleration $\alpha = \frac{M \cdot g \cdot r}{I_A}$

The equation of the motion of the wheel is M.g – T = M.a

Substituting $a = \alpha \,.\, r$ and $I_A = \frac{1}{2} \cdot M \cdot r^2 + M \cdot R^2$ we get

$$T = M \cdot g + \frac{M \cdot g \cdot r^2}{\frac{1}{2} \cdot M \cdot R^2 + M \cdot r^2} = M \cdot g \cdot \left(1 + \frac{2 \cdot r^2}{R^2 + 2 \cdot r^2}\right)$$

Thus for the tension $\frac{T}{2}$ in each string we get

$$\frac{T}{2} = \frac{M \cdot g}{2} \cdot \left(1 + \frac{2 \cdot r^2}{R^2 + 2 \cdot r^2}\right) = \frac{0{,}40 \cdot 9{,}81}{2} \cdot \left(1 + \frac{2 \cdot 9 \cdot 10^{-6}}{3{,}6 \cdot 10^{-3} + 2 \cdot 9 \cdot 10^{-6}}\right) = 1{,}96\ \mathrm{N}$$

$$\boxed{\frac{T}{2} = 1{,}96\ \mathrm{N}}$$

2.4

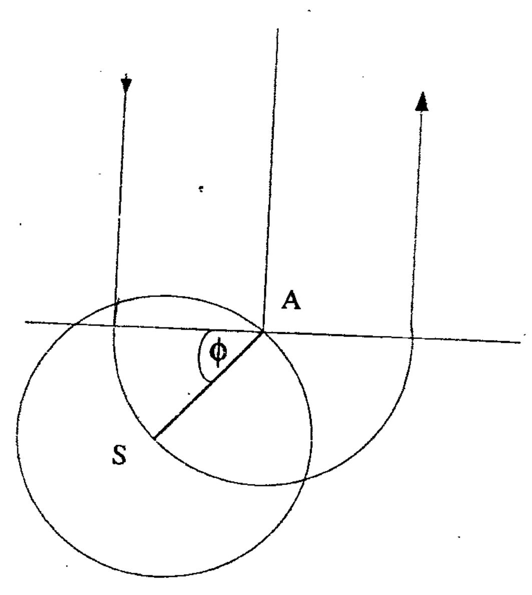

Fig 19.7

After the whole length of the strings is completely unwound, the wheel continues to rotate about A (which is at rest for some interval to be discussed). Let $\dot{\Phi}$ be the angular speed of the centre of mass about the axis through A. The equation of the rotational motion of the wheel about A may be written as $|\tau| = I_A \cdot \ddot{\Phi}$,

where τ is the torque about A, I_A is the moment of inertia about the axis A and $\ddot{\Phi}$ is the angular acceleration about the axis through A.

Hence $M \cdot g \cdot r \cdot \cos\Phi = I_A \cdot \ddot{\phi}$

and $\ddot{\phi} = \frac{M \cdot g \cdot r \cdot \cos\Phi}{I_A}$

Multiplied with $\dot{\Phi}$ gives:

$$\dot{\Phi} \cdot \ddot{\Phi} = \frac{M \cdot g \cdot r \cdot \cos\Phi \cdot \dot{\Phi}}{I_A} \quad \text{or} \quad \frac{1}{2} \cdot \frac{d(\dot{\Phi})^2}{dt} = \frac{M \cdot g \cdot r \cdot \cos\Phi}{I_A} \cdot \frac{d\Phi}{dt}$$

this gives

$$\left(\dot{\Phi}\right)^2 = \frac{2\cdot M\cdot g\cdot r\cdot \sin\Phi}{I_A} + C \qquad \text{[C = arbitrary constant]}$$

If $\Phi = 0$ [s = H] than is $\dot{\Phi} = \omega$

That gives $\omega = \frac{2\cdot M\cdot g\cdot H}{I_A}$ and therefore $C = \frac{2\cdot M\cdot g\cdot H}{I_A}$

Putting these results into the equation above one gets

$$\dot{\Phi} = \omega = \sqrt{\frac{2\cdot M\cdot g\cdot H\cdot \sin\Phi}{I_A}\cdot\left(1+\frac{r}{H}\right)}$$

For $\frac{r}{H} <<< 1$ we get:

$$\omega = \omega'_{MAX} = \sqrt{\frac{2\cdot M\cdot g\cdot H}{I_A}}$$

and

$$v = r\cdot\omega'_{MAX} = r\cdot\sqrt{\frac{2\cdot M\cdot g\cdot H}{I_A}}$$

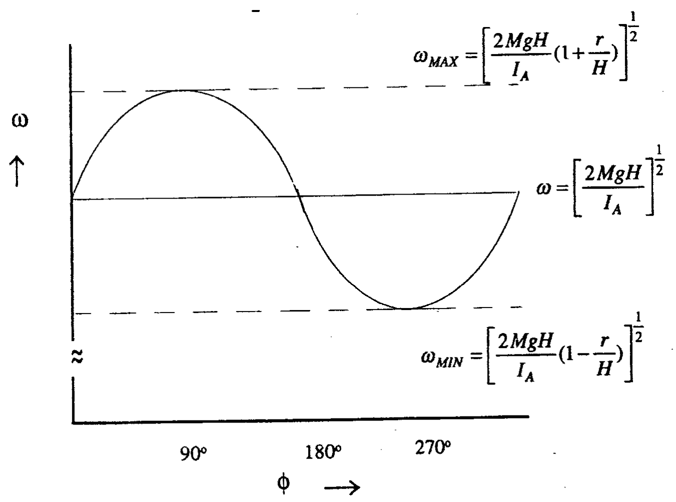

Fig.19.8

Component of the displacement
along x-axis is x = r.sin Φ-r
along y-axis is y = r.cos Φ-r

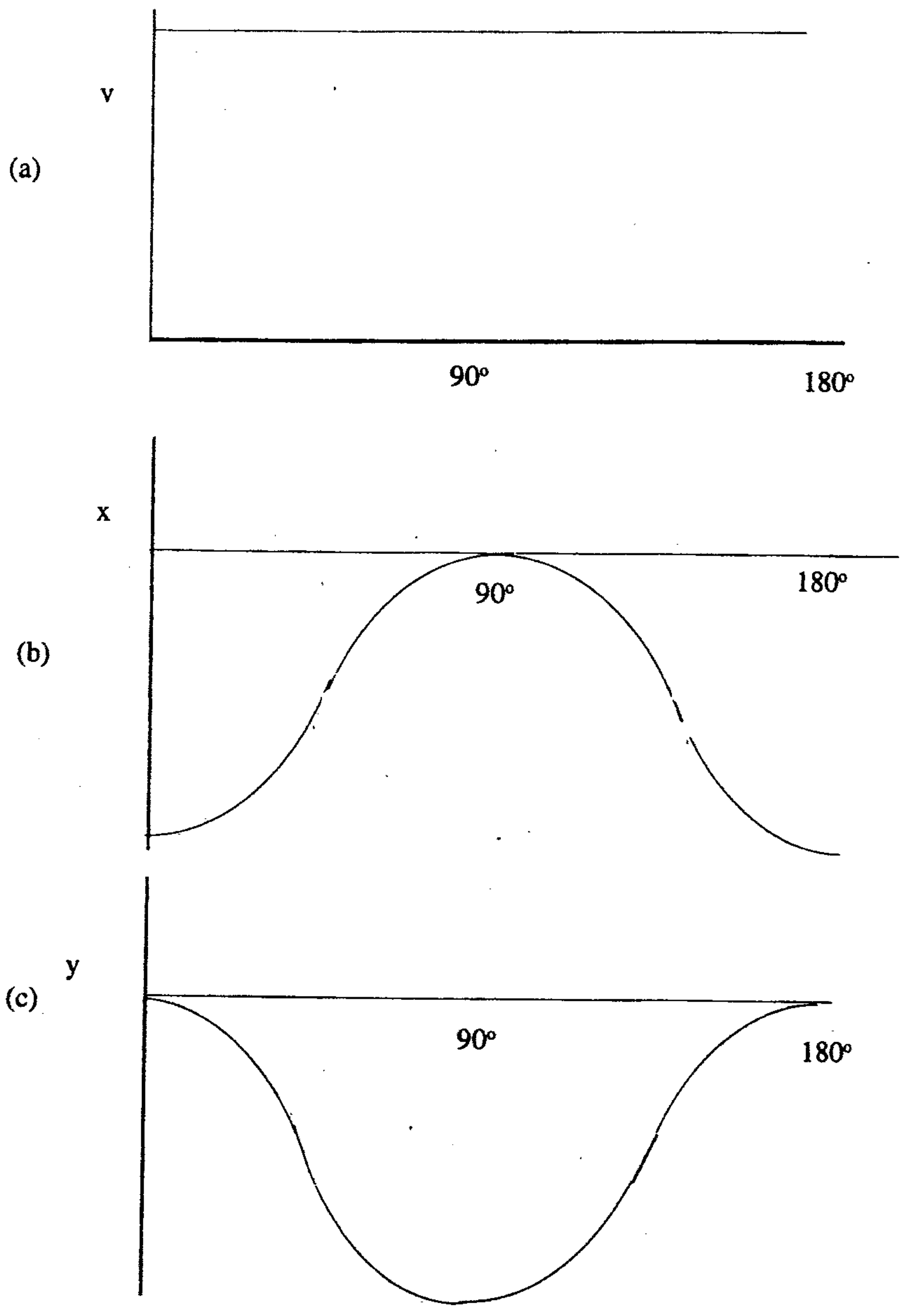

Fig.19.9

2.5

Maximum tension in each string occurs $\dot{\Phi} = \omega'_{MAX}$

The equation of the motion is $T_{MAX} - M \cdot g = M \cdot (\omega'_{MAX})^2 \cdot r$

Putting in T = 20 N and $\omega'_{MAX} = \sqrt{\dfrac{2 \cdot M \cdot g \cdot s}{I_A}}$ (where s is the maximum length of the strings supporting the wheel without breaking) and $I_A = M \cdot \left(\dfrac{R^2}{2} + r^2 \right)$ the numbers one gets:

$20 = 0{,}40 \cdot 9{,}81 \cdot \left(1 + \dfrac{4 \cdot 3 \cdot 10^{-3} \cdot s}{36 \cdot 10^{-4} + 2 \cdot 9 \cdot 10^{-6}} \right)$ This gives: s = 1,24 m

The maximum length of the strings which support maximum tension without breaking is

$\boxed{1{,}24 \text{ m}}$.