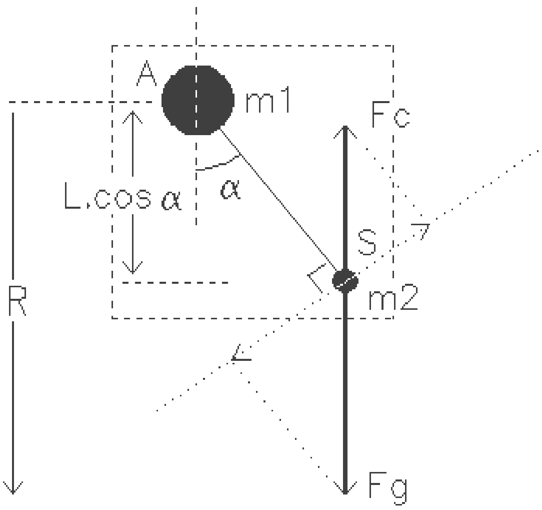
Solution of question 2.

a_1 - Since $m_2 << m_1$, the Atlantis travels around the earth with a constant speed. The motion of the satellite is composed of the circular motion of the Atlantis about the earth and (possibly) a circular motion of the satellite about the Atlantis.
For m_1 we have:

$$m_1.\Omega^2.R = \frac{G.m_1.m_a}{R^2} \rightarrow \Omega^2 = \frac{G.m_a}{R^3}$$



For m_2 we have:

$$m_2.L.\ddot{\alpha} = -(F_g - F_c).\sin(\alpha) = -\left(\frac{G.m_2.m_a}{(R - L.\cos(\alpha))^2} - m_2.\Omega^2.(R - L.\cos(\alpha))\right).\sin(\alpha)$$

Using the approximation:

$$\frac{1}{(R - L.\cos(\alpha))^2} \approx \frac{1}{R^2} + \frac{2.L.\cos(\alpha)}{R^3}$$

and equation (1), one finds:

$$L.\ddot{\alpha} = -\left(\frac{G.m_a}{R^2} + \frac{2.G.m_a}{R^3}.L.\cos(\alpha) - \frac{G.m_a}{R^3}.R + \frac{G.m_a}{R^3}.L.\cos(\alpha)\right).\sin(\alpha)$$

so:

$$\ddot{\alpha} + 3.\Omega^2.\sin(\alpha).\cos(\alpha) = 0 \qquad (2)$$

If α is constant: $\ddot{\alpha} = 0$ --> $\sin(\alpha) = 0$ --> $\alpha = 0$; $\alpha = \pi$
--> $\cos(\alpha) = 0$ --> $\alpha = \pi/2$; $\alpha = 3\pi/2$

a_2 - The situation is stable if the moment $M = m_2.L.\ddot{\alpha}.L = m_2.L^2.\ddot{\alpha}$ changes sign in a manner opposed to that in which the sign of $\alpha - \alpha_0$ changes:

sign($\alpha - \alpha_0$)	- +	- +	- +	- +	- +
α	0	$\pi/2$	π	$3\pi/2$	2π

sign(M)	+ -	- +	+ -	- +	+ -
α	0	$\pi/2$	π	$3\pi/2$	2π

The equilibrium about the angles 0 en π is thus stable, whereas that around $\pi/2$ and $3\pi/2$ is unstable.

b - For small values of α equation (2) becomes:

$$\ddot{\alpha} + 3.\Omega^2.\alpha = 0$$

This is the equation of a simple harmonic motion.
The square of the angular frequency is:

$$\omega^2 = 3.\Omega^2$$

so:

$$\omega = \Omega.\sqrt{3} \quad \rightarrow \quad T_1 = \frac{2\pi}{\omega} = \frac{1}{3}\sqrt{3}.\left(\frac{2\pi}{\Omega}\right) \approx 0{,}58.T_0$$

c_1 - According to Lenz's law, there will be a current from the satellite (S) towards the shuttle (A).

c_2 - For the total energy of the system we have:

$$U = U_{kin} + U_{pot} = \frac{1}{2}.m.\Omega^2.R^2 - \frac{G.m.m_a}{R} = -\frac{1}{2}.\frac{G.m.m_a}{R}$$

A small change in the radius of the orbit corresponds to a change in the energy of:

$$\Delta U = \frac{1}{2}.\frac{G.m.m_a}{R^2}.\Delta R = \frac{1}{2}.m.\Omega^2.R.\Delta R$$

In the situation under c_1 energy is absorbed from the system as a consequence of which the radius of the orbit will decrease.
Is a current source inside the shuttle included in the circuit, which maintains a net current in the opposite direction, energy is absorbed by the system as a consequence of which the radius of the orbit will increase.
From the assumptions in c_2 we have:

$$\Delta U = F_l.v.t = B.I.L.\Omega.R.t = \frac{1}{2}.m.\Omega^2.R.\Delta R \quad \rightarrow \quad t = \frac{1}{2}.\frac{m.\Omega.\Delta R}{B.I.L}$$

Numerical application gives for the time: $t \approx 5{,}8 \,.\, 10^3$ s; which is about the period of the system.

Marking breakdown:

a_1			: 1
a_2			: 1
b	-	Atlantis in uniform circular motion	: 0,5
	-	calculation of the period Ω	: 0,5
	-	equation of motion of the satellite	: 2,5
	-	equation of motion for small angles	: 0,5
	-	period of oscillations	: 1
c_1	-		: 1
c_2	-	calculation of the time the current has to be maintained	: 1,5
	-	increase or decrease of the radius of the orbit	: 0,5

Solution of question 3