Solution Problem 3

Question a:

The velocity v_o of the atoms whose kinetic energy is the mean of the atoms on issuing from the collimator is given is given by:

$$\frac{1}{2}mv_o^2 = \frac{3}{2}kT \Rightarrow v_o = \sqrt{\frac{3kT}{m}} \qquad (1)$$

$$v_o = \sqrt{\frac{3\cdot 1{,}38\cdot 10^{-23}\cdot 10^3}{23\cdot 1{,}67\cdot 10^{-27}}}\ \text{m/s}$$

$v_o \approx 1{,}04\cdot 10^3$ m/s because:

$m \approx 23\ m_p$ (2)

Since this velocity is much smaller than c, $v_o << c$, we may disregard relativistic effects.

Light is made up of photons with energy $h\nu$ and momentum $h\nu/c$.

In the reference system of the laboratory, the energy and momentum conservation laws applied to the absorption process imply that:

$$\frac{1}{2}mv_o^2 + h\nu = \frac{1}{2}mv_1^2 + E;\ mv_o - \frac{h\nu}{c} = mv_1 \Rightarrow \Delta v_1 = v_1 - v_o = \frac{-h\nu}{mc}$$

$$\frac{1}{2}m(v_1^2 - v_o^2) = h\nu - E \Rightarrow \frac{1}{2}m(v_1 + v_o)(v_1 - v_o) = h\nu - E$$

$h\nu/c << mv_o$. Then $v_1 \approx v_o$ and this implies $mv_o\Delta v_1 = h\nu = E$, where we assume that

$v_1 + v_o \approx 2v_o$

Combining these expressions:

$$\nu = \frac{\frac{E}{h}}{1+\frac{v_o}{c}} \qquad (3)$$

and:

$$\Delta v_1 = -\frac{E}{mc}\frac{1}{1+\frac{v_o}{c}} \qquad (4)$$

And substituting the numerical values:

$\nu \approx 5{,}0 \cdot 10^{14}$ Hz $\quad \Delta v_1 \approx -3{,}0 \cdot 10^{-2}$ m/s

If we had analyzed the problem in the reference system that moves with regard to the laboratory at a velocity v_o, we would have that:

$$\frac{1}{2}m(v_1 - v_2)^2 + E = h\nu$$

Where $\nu = \frac{\nu'}{1+\frac{v_o}{c}}$ is the frequency of the photons in the laboratory

system. Disregarding Δv_1^2 we get the same two equations above.

The approximations are justifiable because:

$$-\frac{|\Delta v_1|}{v_o} \sim 10^{-4}$$

Then $v_1 + v_o = 2v_o - \Delta v_1 \approx 2v_o$

Question b:

For a fixed ν:

$$v_o = c\left(\frac{E}{h\nu} - 1\right) \qquad (5)$$

if E has an uncertainty Γ, v_o would have an uncertainty:

$$\Delta v_o = \frac{c\Gamma}{h\nu} = \frac{c\Gamma\left(1+\frac{v_o}{c}\right)}{E} \approx \frac{c\Gamma}{E} = 6{,}25 \text{ m/s} \qquad (6)$$

so the photons are absorbed by the atoms which velocities are in the interval $\left(v_o - \frac{\Delta v_o}{2}, v_o + \frac{\Delta v_o}{2}\right)$

Question c:

The energy and momentum conservation laws imply that:

$$\frac{1}{2}mv_1^2 + E = \frac{1}{2}mv'^2_1 + h\nu'$$

(ν' – is the frequency of emitted photon)

$$mv_1 = mv'_1\cos\varphi + \frac{h\nu'}{c}\cos\theta$$

$$0 = mv'_1\sin\varphi - \frac{h\nu'}{c}\sin\theta$$

The deviation φ of the atom will be greatest when $\theta = \frac{\pi}{2}$, then:

$$mv_1 = mv'_1 \cos\varphi_m; \frac{h\nu'}{c} = mv'_1 \sin\varphi_m \Rightarrow \tan\varphi_m = \frac{h\nu'}{mv_1 c}$$

since $\nu' \approx \nu$:

$$\tan\varphi_m \approx \frac{E}{mv_1 c} \tag{7}$$

$$\varphi_m = \text{arctg}\frac{E}{mvc} \Rightarrow \varphi_m \approx 5 \cdot 10^{-5} \text{ rad} \tag{8}$$

Question d:

As the velocity of the atoms decreases, the frequency needed for resonant absorption increases according to:

$$\nu = \frac{\frac{E}{h}}{1 + \frac{v_o}{c}}$$

When the velocity is $v_o = \Delta v$, absorption will still be possible in the lower part of the level if:

$$h\nu = \frac{E - \frac{\Gamma}{2}}{1 + \frac{v_o - \Delta v}{c}} = \frac{E}{1 + \frac{v_o}{c}} \Rightarrow \Delta v = \frac{c\Gamma}{2E}\left(1 + \frac{v_o}{c}\right) \tag{9}$$

$\Delta v = 3{,}12$ m/s

Question e:

If each absorption-emission event varies the velocity as $\Delta v_1 \approx \frac{E}{mc}$, decreasing velocity from v_o to almost zero would require N events, where:

$$N = \frac{v_o}{|\Delta v_1|} \approx \frac{mcv_o}{E} \Rightarrow N \approx 3{,}56 \cdot 10^4$$

Question f:

If absorption is instantaneous, the elapsed time is determined by the spontaneous emission. The atom remains in the excited state for a certain time, $\tau = \frac{h}{\Gamma}$, then:

$$\Delta t = N\tau = \frac{Nh}{\Gamma} = \frac{mchv_o}{\Gamma E} \Rightarrow \Delta t \approx 3{,}37 \cdot 10^{-9} \text{ s}$$

The distance covered in that time is $\Delta S = v_o \Delta t/2$. Assuming that the motion is uniformly slowed down:

$$\Delta S = \frac{1}{2} mchv_o^2 \Gamma E \Rightarrow \Delta S \approx 1{,}75 \text{ m}$$

Marking Code

a)Finding	v_o	1 pt	Total 3 pt
“	ν	1 pt	
“	Δv_1	1 pt	
b) “	Δv_o	1,5 pt	Total 1,5 pt