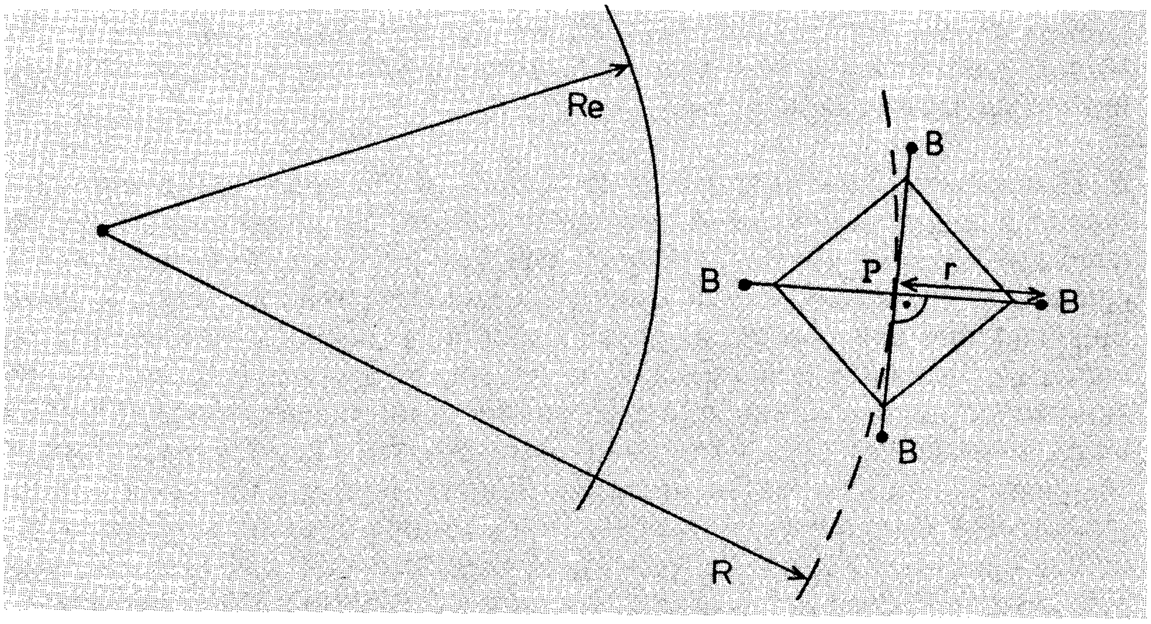
PROBLEM 1 : A ROTATING SATELLITE.

The figure shows a satellite which is circling the Earth in an approximately circular orbit in the Earth's equatorial plane. The satellite consists of a massless central body P and four small peripheral bodies B. The four bodies B each have mass m: they are fastened to P by means of long thin wires of length r that do not stretch. All these five bodies, P and the four bodies B, are coplanar with the equatorial plane, and they can rotate within this plane. The four radial wires are linked to each other by further thin wires which keep the angles between the radial wires constant at 90°.



The link wires are included in the system in order to prevent oscillatory movement of the individual bodies B which would otherwise make the analysis of the movements extremely complicated. All the bodies B rotate around P at the same angular velocity, which is ω with respect to the fixed stars. Thus, the satellite behaves as a rigid body.

Analyze the following questions for the general case, considering all possible situations, including both senses of rotation of the bodies B. Also obtain numerical values for certain of the quantities found in questions (1) and (2)—the quantities required and the necessary numerical data are listed at the end of the problem.

1) The drawing shows the satellite in the position where for the various wires, **r** is parallel, anti-parallel or perpendicular to **R**. (The vector **r** runs from body P to a body B and has length r; the vector **R** runs from the centre of mass of the Earth to the body P.)

   Determine the force exerted by a radial wire on one of the bodies B in each of these four positions. These positions correspond approximately to the maximum and minimum forces.

2) Inside the four bodies B there are four identical machines, powered by solar energy, connected to the radial wires. Each machine pulls the wire in, towards B, for a short time whenever there is near maximum force in the wire (as indicated in the previous question), and lets the same length of wire out again when the tension is at a minimum. The length of wire that is pulled in and let out is 1% of the mean length of the radial wire. The mean length does not change with time.

   What is the net power converted by one machine, averaged over one rotation of the satellite?

   The net power is defined as $\frac{W_1 - W_2}{T}$, where W_1 is the work that the machine performs on the wire when pulling it in, W_2 is the work that the wire performs on the machine when it is reeled out and T is the period of rotation.

3) Discuss the changes in the motion of the satellite that are caused by the action of the machines. In particular, analyze any changes that may occur in each of the situations listed in the table overleaf.

   Fill in the table with your results and comments, and don't forget to hand it in.

Data:

Numerical answers are required in the following situation:

The radius of the orbit of the central body is given by $R = R_E + 500\,\text{km}$.

The mean length of the radial wires is $r = 100\,\text{km}$.
Thus the diameter of the satellite system is 200 km.

The bodies B have masses $m = 1000\,\text{kg}$.

Initially the four bodies B rotate, as referred to the stars, around the central body P at 10 revolutions/hour.

The masses of the wires are negligible, and the central body P is massless.

Advice:

Consider both senses of rotation for ω.
Exact solutions are not expected. Results with 5% accuracy are fully acceptable.
Ignore the gravitational effect of the moon and the sun.

Useful data:

Mass of Earth	$M_E = 5.97 \times 10^{24}\,\text{kg}$
Gravitational constant	$G = 6.673 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$
Radius of Earth at equator	$R_E = 6378\,\text{km}$
Denote the product $M_E G$ by K.	$K = 3.983 \times 10^{14}\,\text{m}^3\,\text{s}^{-2}$

Country code:

ANSWER TABLE

Fill in this table as part of your answer. Write down equalities or inequalities and/or short explanations where necessary.

The quantity indicated below ...	increases if ...	decreases if ...	stays unchanged if ...	stays unchanged in all situations.
orbital **velocity** of the satellite				Yes ☐ No ☐
radius R of the orbit of the satellite				Yes ☐ No ☐
angular velocity ω of the satellite				Yes ☐ No ☐
gravitational **potential energy** of the satellite				Yes ☐ No ☐

Could the satellite reach a higher orbit as a result of the work done by the machines? Yes ☐ No ☐
Could the satellite reach an arbitrarily high orbit, practically leaving the gravitational influence of Earth? Why? Answer:..