

What is the new equilibrium temperature of the satellite? The exact answer is not needed. Therefore, do not perform any tedious integrations; make approximations where necessary. The integral over the entire range is

$$\int_0^{\infty} \frac{\eta^3 d\eta}{e^{\eta} - 1} = \frac{\pi^4}{15}$$

and the maximum of $\eta^3/(e^{\eta} - 1)$ occurs at $\eta \approx 2.82$. For small η you can expand the exponential function as $e^{\eta} \approx 1 + \eta$.

3) If we now have a real satellite, with extending solar panels that generate electricity, the dissipated heat in the electronics inside the satellite acts as an extra source of heat. Assuming that the power of the internal heat source is 1 kW, what is the equilibrium temperature of the satellite in case 2 above?

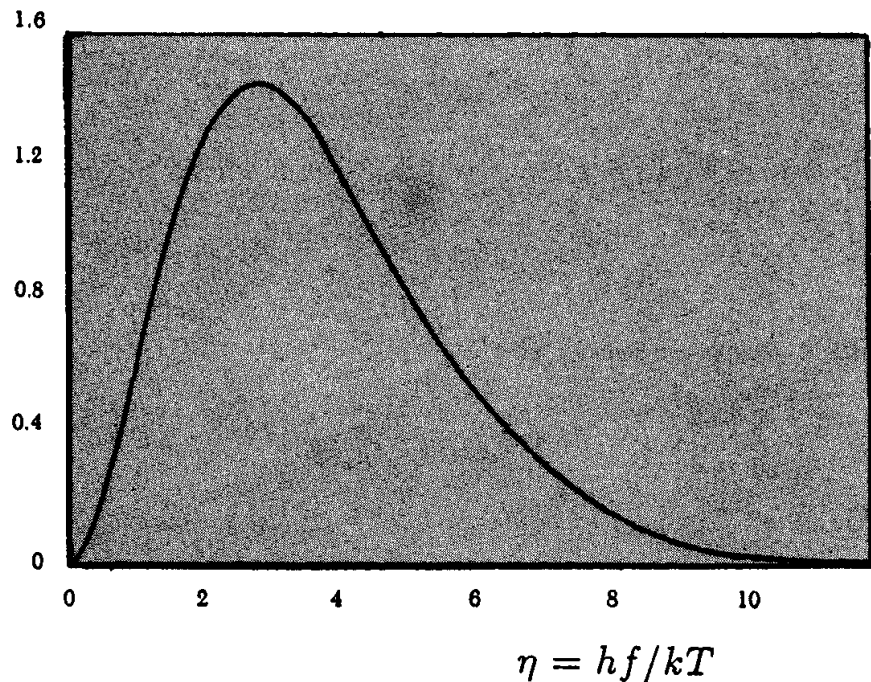
4) A manufacturer advertises a special paint in the following way:

"This paint will reflect more than 90% of all incoming radiation (both visible light and infrared) but it will radiate at all frequencies (visible light and infrared) as a black body, thus removing lots of heat from the satellite. Thus the paint will help keep the satellite as cool as possible."

Can such paint exist? Why or why not?

5) What properties should a coating have in order to raise the temperature of a spherical body similar to that of the satellite considered here above the temperature calculated in 1?

$$\frac{c^3 h^3}{8\pi k^4} \frac{u(T, f)}{T^4}$$



SOLUTION : PROBLEM 3

1. Over the whole surface of the sun, the emitted energy is $4\pi R_{Sun}^2 \cdot \sigma T_{Sun}^4$. All this energy passes through a spherical shell at earth's distance R , where the intensity now is $4\pi R_{Sun}^2 \cdot \sigma T_{Sun}^4 / 4\pi R^2$.

The satellite is a circular object absorbing

$$\pi r_{sat}^2 \cdot \frac{4\pi R_{Sun}^2 \cdot \sigma T_{Sun}^4}{4\pi R^2}$$

but a spherical object emitting $4\pi r_{sat}^2 \cdot \sigma T_{sat}^4$.

Equating this absorption and emission we get $T_{sat} = T_{Sun} \sqrt{\frac{R_{Sun}}{2R}}$ giving $T_{sat} = 289 \text{ K} = 16 \text{ }^\circ\text{C}$.

2. We have to calculate what part of the absorbed power comes from the part of the spectrum below 1200 K.

$$\eta_{cutoff} = \frac{1200 \text{ K}}{6000 \text{ K}} = 0.2 \ll 1$$

This fraction of power is

$$\begin{aligned} \delta &= \int_0^{\eta_{cutoff}} \frac{\eta^3 d\eta}{e^\eta - 1} / \int_0^{\eta_\infty} \frac{\eta^3 d\eta}{e^\eta - 1} \\ &\approx \int_0^{\eta_{cutoff}} \eta^2 d\eta / \frac{\pi^4}{15} = \frac{\eta_{cutoff}^3}{3} / \frac{\pi^4}{15} = 4.1 \cdot 10^{-4} \end{aligned}$$

Now, the satellite is cold with respect to 1200 K so we ignore that a small part of the satellite blackbody emission will be retained. The energy balance is now

$$4\pi r_{sat}^2 \cdot \sigma T_{sat}^4 = \delta \cdot \pi r_{sat}^2 \cdot \frac{4\pi R_{Sun}^2 \cdot \sigma T_{Sun}^4}{4\pi R^2}$$

by which the new satellite temperature is the previous corrected by a factor $\delta^{1/4}$

$$T_{sat} = (4.1 \cdot 10^{-4})^{1/4} \cdot 289 \text{ K} = 41 \text{ K}$$

3. The whole absorbed energy is

$$\delta \cdot \pi r_{sat}^2 \cdot \frac{4\pi R_{Sun}^2 \cdot \sigma T_{Sun}^4}{4\pi R^2} = 0.5 \text{ W}$$

which is small (ignorable) compared to $P_{internal} = 1 \text{ kW}$. Thus the energy balance becomes

$$P_{internal} = 4\pi r_{sat}^2 \cdot \sigma T_{sat}^4$$

PROBLEM **3** : A SATELLITE IN SUNSHINE

giving $T_{sat} = 274$ K ($\eta = 4.38$). Note: strictly speaking, this is not accurate, because for a blackbody radiation of 274 K, some 33 % of the emitted power lies above the 1200 K cutoff ! This means that the satellite has to be hotter, to emit all of the 1 kW in frequencies below the cutoff. The resulting integral equation is

$$\left(\frac{\eta}{4.38}\right)^4 = \int_0^{\eta} \frac{\eta^3 d\eta}{e^{\eta} - 1} / \frac{\pi^4}{15}$$

which can be solved numerically by iteration. The true solution is $\eta = 3.80$ corresponding to a temperature of 316 K.

4. The paint cannot exist, because it would violate the second law of thermodynamics. The physics textbook explanation is the principle of detailed balance, which means that for equilibrium to exist, the emission and absorption in a given frequency interval must match exactly. This should not be confused with the fact that reflection and absorption can be quite different. If the manufacturer's paint existed, one could create a temperature difference between two bodies in a closed system, and hence a perpetum mobile.
5. The coating should be transparent for high frequencies (in the range of the peak or tail of the sun's radiation), but reflective and hence insulating at low frequencies (the satellite temperature).