Theoretical Problem 1

RELATIVISTIC PARTICLE

In the theory of special relativity the relation between energy E and momentum P or a free particle with rest mass m_0 is

$$E = \sqrt{p^2c^2 + m_0^2c^4} = mc^2$$

When such a particle is subject to a conservative force, the total energy of the particle, which is the sum of $\sqrt{p^2c^2 + m_0^2c^4}$ and the potential energy, is conserved. If the energy of the particle is very high, the rest energy of the particle can be ignored (such a particle is called an ultra relativistic particle).

1) consider the one dimensional motion of a very high energy particle (in which rest energy can be neglected) subject to an attractive central force of constant magnitude f. Suppose the particle is located at the centre of force with initial momentum p_0 at time t=0. Describe the motion of the particle by separately plotting, for at least one period of the motion: x against time t, and momentum p against space coordinate x. Specify the coordinates of the "turning points" in terms of given parameters p_0 and f. Indicate, with arrows, the direction of the progress of the mothon in the (p, x) diagram. There may be short intervals of time during which the particle is not ultrarelativistic. However, these should be neglected.

   Use Answer Sheet 1.

2) A meson is a particle made up of two quarks. The rest mass M of the meson is equal to the total energy of the two-quark system divided by c^2.

   Consider a one--dimensional model for a meson at rest, in which the two quarks are assumed to move along the x-axis and attract each other with a force of constant magnitude f It is assumed they can pass through each other freely. For analysis of the high energy motion of the quarks the rest mass of the quarks can be neglected. At time t=0 the two quarks are both at x=0. Show separately the motion of the two quarks graphically by a (x, t) diagram and a (p, x) diagram, specify the coordinates of the "turning points" in terms of M and f, indicate the direction of the process in your (p, x) diagram, and determine the maximum distance between the two quarks.

   Use Answer Sheet 2.

3) The reference frame used in part 2 will be referred to as frame S, the Lab frame, referred to as S, moves in the negative x-direction with a constant velocity v=0.6c. the coordinates in the two reference frames are so chosen that the point

x=0 in S coincides with the point $x'=0$ in S'' at time $t=t'=0$. Plot the motion of the two quarks graphically in a (x', t') diagram. Specify the coordinates of the turning points in terms of M, f and c, and determine the maximum distance between the two quarks observed in Lab frame S'.

Use Answer Sheet 3.

The coordinates of particle observed in reference frames S and S'' are related by the Lorentz transformation

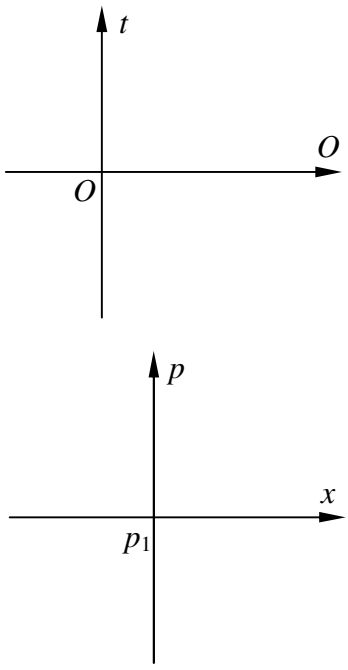
$$\begin{cases} x' = \gamma(x + \beta ct) \\ t' = \gamma(t + \beta \dfrac{x}{c}) \end{cases}$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$ and v is the velocity of frame S moving relative to the frame S''.

4) For a meson with rest energy Mc^2=140 MeV and velocity 0.60c relative to the Lab frame S'', determine its energy E' in the Lab Frame S''.

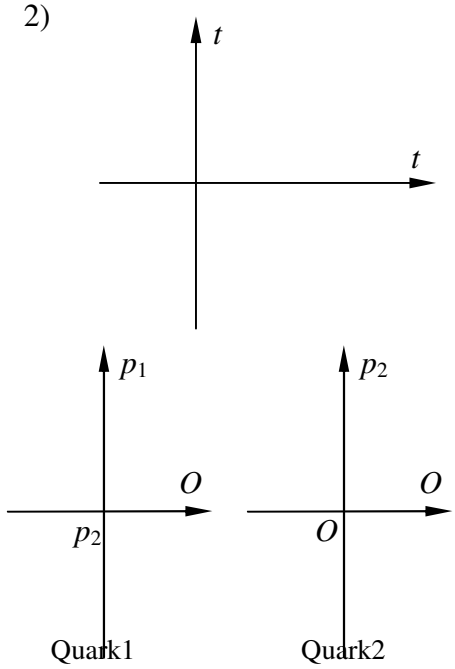
ANSWER SHEET 1

1)



ANSWER SHEET 2

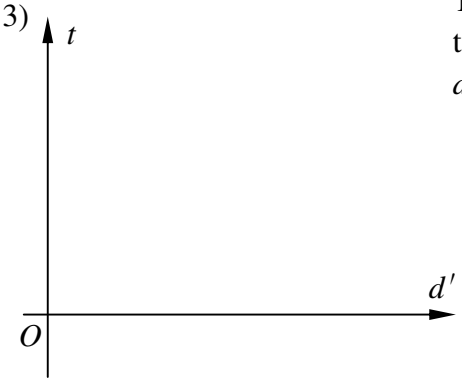
2)



The maximum distance between the two quarks is d=

ANSWER SHEET 3

3)



The maximum distance between the two quarks observed in S′ frame is

$d' =$