Theoretical Problem 1—Solution

1) **1a. Taking the force center as the origin of the space coordinate** x and the zero potential point, the potential energy of the particle is

$$U(x) = f\,|x| \tag{1}$$

The total energy is

$$W = \sqrt{p^2c^2 + m_0^2c^4} + f\,|x|.$$

1b. Neglecting the rest energy, we get

$$W = |p|c + f\,|x|, \tag{2}$$

Since W is conserved throughout the motion, so we have

$$W = |p|c + f\,|x| = p_0 c, \tag{3}$$

Let the x axis be in the direction of the initial momentum of the particle,

$$\left.\begin{array}{lll} pc + fx = p_0c & \text{when} & x>0, \quad p>0; \\ -pc + fx = p_0c & \text{when} & x>0, \quad p<0; \\ pc - fx = p_0c & \text{when} & x<0, \quad p>0; \\ -pc - fx = p_0c & \text{when} & x<0, \quad p<0. \end{array}\right\} \tag{4}$$

The maximum distance of the particle from the origin, let it be L, corresponds to p=0. It is

$$L = p_0 c / f\ .$$

1c. From Eq. 3 and Newton's law

$$\frac{dp}{dt} = F = \begin{cases} -f, & x>0; \\ f, & x<0; \end{cases} \tag{5}$$

we can get the speed of the particle as

$$\left|\frac{dx}{dt}\right| = \frac{c}{f}\left|\frac{dp}{dt}\right| = c, \tag{6}$$

i.e. the particle with very high energy always moves with the speed of light except that it is in the region extremely close to the points $x = \pm L$. The time for the particle to move from origin to the point $x = L$, let it be denoted by τ, is

$$\tau = L / c = p_0 / f .$$

So the particle moves to and for between $x = L$ and $x = -L$ with speed c and period $4\tau = 4p_0 / f$. The relation between x and t is

$$\left.\begin{array}{ll} x = ct, & 0 \le t \le \tau \\ x = 2L - ct, & \tau \le t \le 2\tau, \\ x = 2L - ct, & 2\tau \le t \le 3\tau, \\ x = ct - 4L, & 3\tau \le t \le 4\tau, \end{array}\right\} \tag{7}$$

The required answer is thus as given in Fig. 1 and Fig. 2.

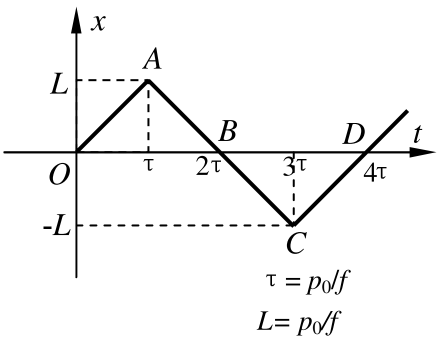

Fig. 1

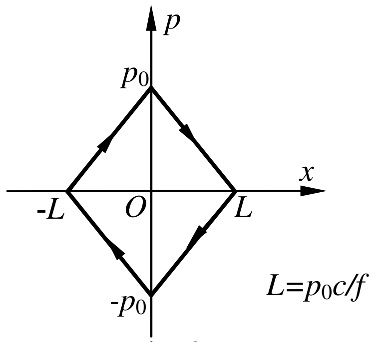

Fig. 2

2) The total energy of the two-quark system can be expressed as

$$Mc^2 = |p_1|c + |p_2|c + f|x_1 - x_2|, \tag{8}$$

where x_1, x_2 are the position coordinates and p_1, p_2 are the momenta of quark 1 and quark 2 respectively. For the rest meson, the total momentum of the two quarks is zero and the two quarks move symmetrically in opposite directions, we have

$$p = p_1 + p_2 = 0 , \qquad p_1 = -p_2 , \quad x_1 = -x_2 . \tag{9}$$

Let p_0 denote the momentum of the quark 1 when it is at x=0, then we have

$$Mc^2 = 2p_0 c \qquad \text{or} \qquad p_0 = Mc / 2 \tag{10}$$

From Eq. 8, 9 and 10, the half of the total energy can be expressed in terms of p_1 and x_1 of quark 1:

$$p_0 c = |p_1|c + f|x_1|, \tag{11}$$

just as though it is a one particle problem as in part 1 (Eq. 3) with initial momentum

$p_0 = Mc/2$. From the answer in part 1 we get the (*x*, *t*) diagram and (*p*, *x*) diagram of the motion of quark 1 as shown in Figs. 3 and 4. For quark 2 the situation is similar except that the signs are reversed for both *x* and *p*; its (*x*, *t*) and (*p*, *x*) diagrams are shown in Figs. 3 and 4.

The maximum distance between the two quarks as seen from Fig. 3 is

$$d = 2L = 2p_0c/f = Mc^2/f\ . \tag{12}$$

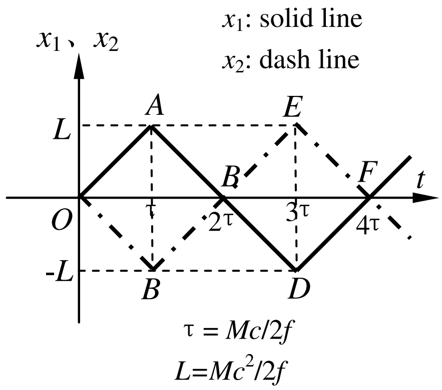

Fig. 3

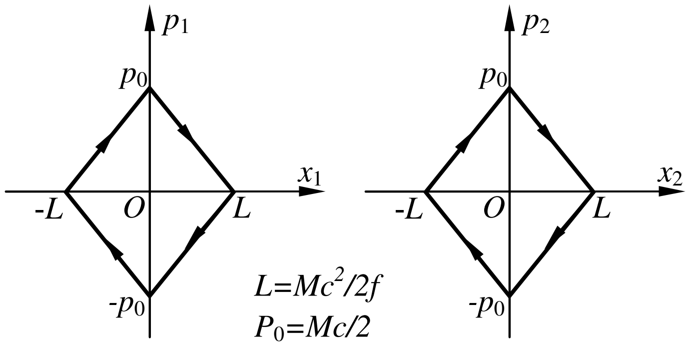

Fig. 4a Quark1

Fig. 4b Quark2

3) The reference frame *S* moves with a constant velocity *V*=0.6*c* relative to the Lab frame S'' in the x' axis direction, and the origins of the two frames are coincident at the beginning ($t = t' = 0$). The Lorentz transformation between these two frames is given by:

$$\begin{aligned} x' &= \gamma(x + \beta ct), \\ t' &= \gamma(t + \beta x/c), \end{aligned} \tag{13}$$

where $\beta = V/c$, and $\gamma = 1/\sqrt{1-\beta^2}$. With $V = 0.6c$, we have $\beta = 3/5$, and $\gamma = 5/4$. Since the Lorenta transformation is linear, a straight line in the (*x*, *t*) diagram transforms into a straight line the (x', t') diagram, thus we need only to calculate the coordinates of the turning points in the frame S'.

For quark 1, the coordinates of the turning points in the frames S and S' are as follows:

Frame S		Frame S'	
x_1	t_1	$x_1' = \gamma(x_1 + \beta c t_1)$ $= \frac{5}{4}x_1 + \frac{3}{4}ct_1$	$t_1' = \gamma(t_1 + \beta x_1 / e)$ $= \frac{5}{4}t_1 + \frac{3}{4}x_1 / c$
0	0	0	0
L	τ	$\gamma(1+\beta)L = 2L$	$\gamma(1+\beta)\tau = 2\tau$
0	2τ	$2\gamma\beta L = \frac{3}{2}L$	$2\gamma\tau = \frac{5}{2}\tau$
$-L$	3τ	$\gamma(3\beta - 1)L = L$	$\gamma(3-\beta)\tau = 3\tau$
0	4τ	$4\gamma\beta L = 3L$	$4\gamma\tau = 5\tau$

where $L = p_0 c / f = Mc^2 / 2f$, $\tau = p_0 / f = Mc / 2f$.

For quark 2, we have

Frame S		Frame S'	
x_2	t_2	$x_2' = \gamma(x_2 + \beta c t_2)$ $= \frac{5}{4}x_2 + \frac{3}{4}ct_2$	$t_2' = \gamma(t_2 + \beta x_2 / c)$ $= \frac{5}{4}t_2 + \frac{3}{4}x_2 / c$
0	0	0	0
$-L$	τ	$-\gamma(1-\beta)L = -\frac{1}{2}L$	$\gamma(1-\beta)\tau = \frac{1}{2}\tau$
0	2τ	$2\gamma\beta L = \frac{3}{2}L$	$2\gamma\tau = \frac{5}{2}\tau$
L	3τ	$\gamma(3\beta + 1)L = \frac{7}{2}L$	$\gamma(3+\beta)\tau = \frac{9}{2}\tau$
0	4τ	$4\gamma\beta L = 3L$	$4\gamma\tau = 5\tau$

With the above results, the (x', t') diagrams of the two quarks are shown in Fig. 5.

The equations of the straight lines *OA* and *OB* are:

$$x_1'(t') = ct'; \qquad 0 \le t' \le \gamma(1+\beta)\tau = 2\tau; \tag{14a}$$

$$x_2'(t') = -ct'; \qquad 0 \le t' \le \gamma(1-\beta)\tau = \frac{1}{2}\tau \tag{14b}$$

The distance between the two quarks attains its maximum d' when $t' = \frac{1}{2}\tau$, thus we have maximum distance

$$d' = 2c\gamma(1-\beta)\tau = 2\gamma(1-\beta)L = \frac{Mc^2}{2f}. \qquad (15)$$

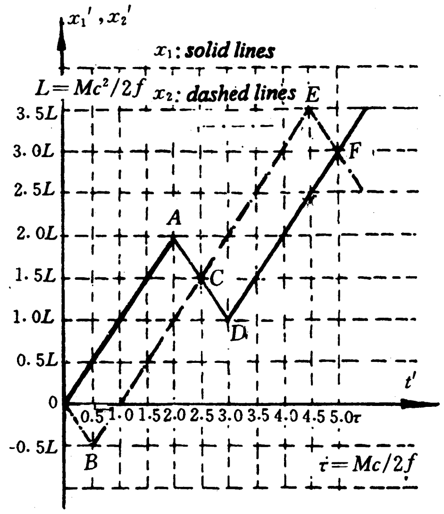

Fig. 5

4) It is given the meson moves with velocity V=0.6 $c$relative to the Lab frame, its energy measured in the Lab frame is

$$E' = \frac{Mc^2}{\sqrt{1-\beta^2}} = \frac{1}{0.8}\times 140 = 175\,\text{MeV}.$$

Grading Scheme

Part 1 2 points, distributed as follows:

0.4 point for the shape of $x(t)$ in Fig. 1;

0.3 point for 4 equal intervals in Fig. 1;

(0.3 for correct derivation of the formula only)

0.1 each for the coordinates of the turning points A and C, 0.4 point in total;

0.4 point for the shape of $p(x)$ in fig. 2; (0.2 for correct derivation only)

0.1 each for specification of p_0, $L = p_0 c / f$, $-p_0$, $-L$ and arrows, 0.5 point in total.

(0.05 each for correct calculations of coordinate of turning points only).

Part 2 4 points, distributed as follows:

0.6 each for the shape of $x_1(t)$ and $x_2(t)$, 1.2 points in total;

0.1 each for the coordinates of the turning points A, B, D and E in Fig. 3, 0.8 point in total;

0.3 each for the shape of $p_1(x_1)$ and $p_2(x_2)$, 0.6 point in total;

0.1 each for $p_0 = Mc/2$, $L = Mc^2/2f$, $-p_0$, $-L$ and arrows in Fig. 4a and Fig. 4b, 1 point in total;

0.4 point for $d = Mc^2/f$

Part 3 3 point, distributed as follows:

0.8 each for the shape of $x_1'(t')$ and $x_2'(t')$, 1.6 points in total;

0.1 each for the coordinates of the turning points A, B, D and E in Fig. 5, 0.8 point in total; (0.05 each for correct calculations of coordinate of turning points only).

0.6 point for $d' = Mc^2/2f$.

Part 4 1 point (0.5 point for the calculation formula; 0.5 point for the numerical value and units)