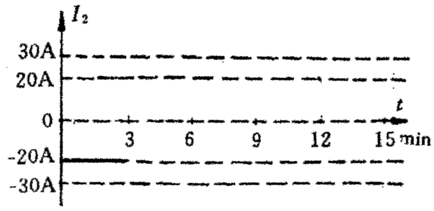


5d

Theoretical Problem 2—Solution

1) For $t=t_1$ to t_3

Since $r=0$, the voltage across the magnet $V_M = LdI_1/dt$ =0, therefore,

$$I_1 = I_1(t_1) = \frac{1}{2}I_0;$$

$$I_2 = I - I_1 = I - \frac{1}{2}I_0.$$

For $t=t_3$ to t_4

Since I_2=0 at $t=t_3$, and I is kept at $\frac{1}{2}I_0$ after

$t = t_3$, $V_M = I_2 r_n = 0$, therefore, I_1 and I_2 will not change.

$$I_1 = \frac{1}{2}I_0;$$

$$I_2 = 0$$

These results are shown in Fig. 6.

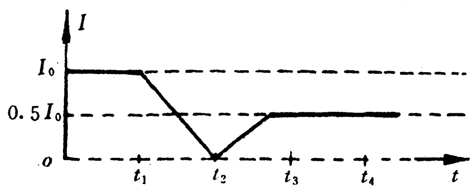
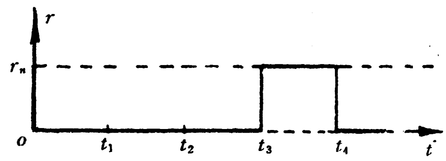
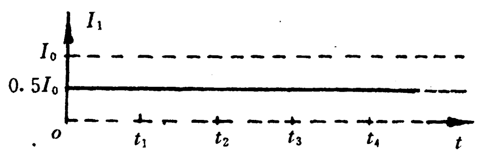


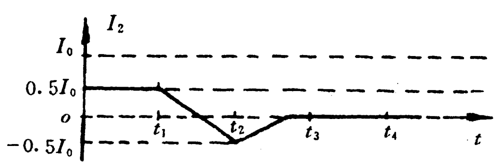
Fig. 6a



6b



6c



6d

2) For $t=0$ to $t=1\,\text{min}$:

Since $r=0$, $V_M = LdI_1/dt = 0$

$$I_1 = I_1(0) = 0$$

$$I_2 = I - I_1 = 0.5\,\text{A}.$$

At $t=1\,\text{min}$, r suddenly jumps from O to r_n, I will drop from E/R to $E/(R+r_n)$ instantaneously, because I_1 can not change abruptly due to the inductance of the magnet coil. For E/R=0.5A, $R=7.5\Omega$ and $R_n=5\Omega$. I will drop to 0.3A.

For $t=1$ min to 2 min:

I, I_1 and I_2 gradually approach their steady values:

$$I = \frac{E}{R} = 0.5\,\text{A},$$

$$I_1 = I = 0.5\,\text{A}$$

$$I_2 = 0.$$

The time constant

$$\tau = \frac{L(R+r_n)}{Rr_n}.$$

When $L=10\,\text{H}$, $R=7.5\Omega$ and $r_n=5\Omega$, $\tau=3\,\text{sec}$.

For $t=2\,\text{min}$ to 3 min:

Since $r=0$, I_1 and I_2 will not change, that is

$$I_1 = 0.5\,\text{A and } I_2 = 0$$

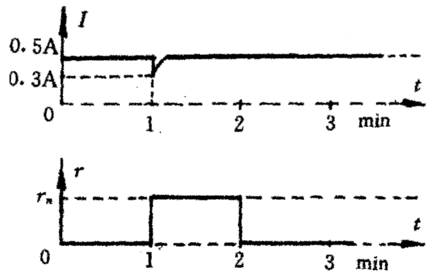
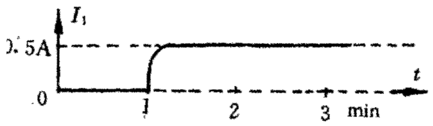
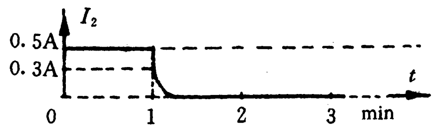


Fig. 7a

7b



7c



7d

3) The operation steps are:

First step

Turn on power switch *K*, and increase the total current *I* to 20 A, i. e. equal to I_1. Since the superconducting switch is in the state $r = 0$, so that $V_M = L \quad dI_1 / dt = 0$, that is, I_1 can not change and I_2 increases by 20A, in other words, I_2 changes from -20 A to zero.

Second step

Switch r from 0 to r_n.

Third step

Gradually reduce I to zero while keeping $I_2 < 0.5$ A: since $I_2 = V_M / r_n$ and $V_m = L\, dI_1 / dt$, when $L = 10$ H, $r_n = 5\Omega$, the requirement $I_2 < 0.5$ A corresponds to $dI_1 / dt < 0.25$ A/sec, that is, a drop of <15A in 1 min. In Fig. 8 $dI / dt \sim 0.1$A/sec and dI_1 / dt is around this value too, so the requirement has been fulfilled.

Final step

Switch r to zero when $V_M = 0$ and turn off the power switch *K*. These results are shown in Fig. 8.

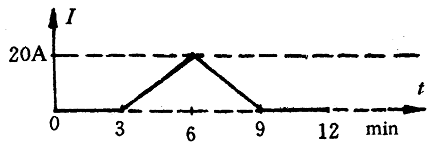
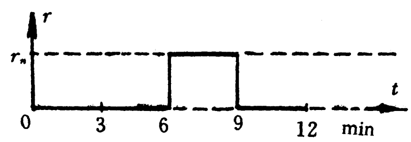
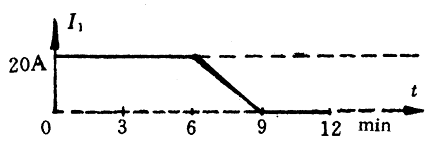


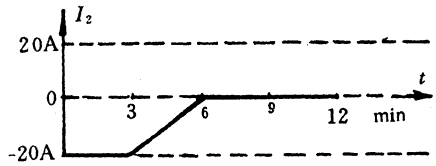
Fig. 8a



8b



8c



8d

4) **First step** and **second step** are the same as that in part 3, resulting in $I_2 = 0$.

Third step Increase I by 10 A to 30 A with a rate subject to the requirement $I_2 < 0.5$ A.

Fourth step Switch r to zero when $V_M = 0$.

Fifth step Reduce I to zero, $I_1 = 30$ A will not change because V_M is zero. $I_2 = I - I_1$ will change to -30 A. The current flowing through the magnet is thus closed by the superconducting switch.

Final step Turn off the power switch *K*. The magnet is operating in the persistent mode.

These results are shown in Fig. 9.

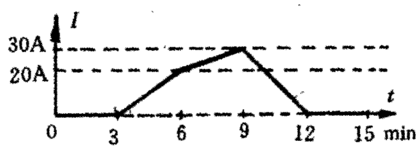
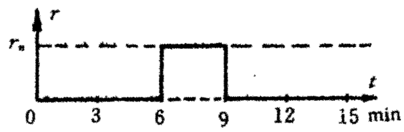
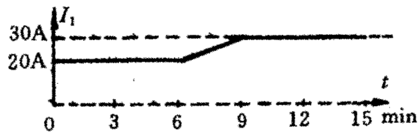


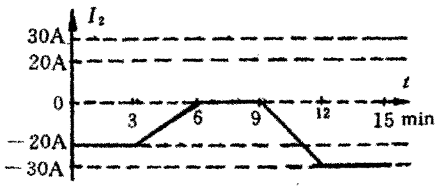
Fig. 9a



9b



9c



9d

Grading Scheme

Part 1, 2 points:

0.5 point for each of I_1, I_2 from $t = t_1$ to t_3 and I_1, I_2 from $t = t_3$ to t_4.

Part 2, 3 points:

0.3 point for each of I_1, I_2 from $t = 0$ to 1 min, I, I_1, I_2 at $t = 1$ min, and I_0, I_1, I_2 from $t=1$ to 2 min;

0.2 point for each of I, I_1, and I_2 from $t=2$ to 3 min.

Part 3, 2 points:

0.25 point for each section in Fig. 8 from $t=3$ to 9 min, 8 sections in total.

Part 4, 3 points:

0.25 point for each section in Fig. 9 from $t=3$ to 12 min, 12 sections in total.