Theoretical Problem 3—Solution

1) When disc A collides with disc B, let n be the unit vector along the normal to the surfaces at the point of contact and t be the tangential unit vector as shown in the figure. Let φ be the angle between n and the x axis. Then we have

$$b = (R_A + R_B)\sin\varphi$$

The momentum components of A and B along n and t before collision are:

$$mV_{An} = mV\cos\varphi,\ mV_{Bn} = 0,$$

$$mV_{At} = mV\sin\varphi,\ mV_{Bt} = 0.$$

Denote the corresponding momentum components of A and B after collision by mV'_{An}, mV'_{Bn}, mV'_{At}, and mV'_{Bt}. Let ω_A and ω_B be the angular velocities of A and B about the axes through their centers after collision, and I_A and I_B be their corresponding moments of intertia. Then,

$$I_A = \frac{1}{2}mR_A^2, \qquad I_B = \frac{1}{2}mR_B^2$$

The conservation of momentum gives

$$mV\cos\varphi = mV'_{An} + mV'_{Bn}, \tag{1}$$

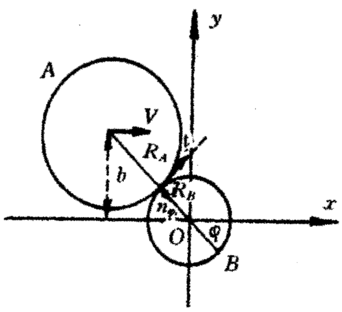
$$mV\sin\varphi = mV'_{At} + mV'_{tn}, \tag{2}$$

The conservation of angular momentum about the axis through O gives

$$mVb = mV'_{At}(R_A + R_B) + I_A\omega_A + I_B\omega_B \tag{3}$$

The impulse of the friction force exerted on B during collision will cause a momentum change of mV'_{At} along t and produces an angular momentum $I_B\omega_B$ simultaneously. They are related by.

$$mV'_{Bt}R_b = I_B\omega_B \tag{4}$$



During the collision at the point of contact A and B acquires the same tangential velocities, so we have

$$V'_{At} - \omega_A R_A = V'_{Bt} - \omega_B R_B \tag{5}$$

It is given that the magnitudes of the relative velocities along the normal direction of the two discs before and after collision are equal, i. e.

$$V\cos\varphi = V'_{Bn} - V'_{An}\,. \tag{6}$$

From Eqs. 1 and 6 we get

$$V'_{An} = 0\,,$$

$$V'_{Bn} = V\cos\varphi\,.$$

From Eqs. 2 to 5, we get

$$V'_{At} = \frac{5}{6}V\sin\varphi\,,$$

$$V'_{Bt} = \frac{1}{6}V\sin\varphi\,,$$

$$\omega_A = \frac{V\sin\varphi}{3R_A}\,,$$

$$\omega_B = \frac{V\sin\varphi}{3R_B}\,.$$

The x and y components of the velocities after collision are:

$$V'_{Ax} = V'_{An}\cos\varphi + V'_{At}\sin\varphi = \frac{5Vb^2}{6(R_A+R_B)^2}\,, \tag{7}$$

$$V'_{Ay} = -V'_{An}\sin\varphi + V'_{At}\cos\varphi = \frac{5Vb\sqrt{(R_A+R_B)^2-b^2}}{6(R_A+R_B)^2}\,, \tag{8}$$

$$V'_{Bx} = V'_{Bn}\cos\varphi + V'_{Bt}\sin\varphi = \left[1 - \frac{5b^2}{6(R_A+R_B)^2}\right], \tag{9}$$

$$V'_{By} = -V'_{Bn}\sin\varphi + V'_{Bt}\cos\varphi = -\frac{5Vb\sqrt{(R_A+R_B)^2-b^2}}{6(R_A+R_B)^2}\,, \tag{10}$$

2) After the collision, the kinetic energy of disc A is

$$E'_A = \frac{1}{2}m(V'^2_{Ax} + V'^2_{Ay}) + \frac{1}{2}I_A\omega_A^2 = \frac{3mV^2b^2}{8(R_A+R_B)^2} \tag{11}$$

while the kinetic energy of disc B is

$$E'_B = \frac{1}{2}m(V'^2_{Bx} + V'^2_{By}) + \frac{1}{2}I_B\omega_B^2 = \frac{1}{2}mV^2\left[1 - \frac{11b^2}{12(R_A + R_B)^2}\right] \qquad (12)$$

Grading Scheme

1. After the collision, the velocity components of discs A and B are shown in Eq. 7, 8, 9 and 10 of the solution respectively. The total points of this part is 8. 0. If the result in which all four velocity components are correct has not been obtained, the point is marked according to the following rules.

0.8 point for each correct velocity component;

0.8 point for the correct description of that the magnitudes of the relative velocities of the discs along the line joining their centers are the same before and after the collision.

0.8 point for the correct description of the conservation for angular momentum;

0.8 point for the correct description of the equal tangential velocity at the touching point;

0.8 point for the correct description of the relation between the impulse and the moment of the impulse.

2. After the collision, the kinetic energies of disc A and disc B are shown in Eqs. 11 and 12 of the solution respectively.

1.0 point for the correct kinetic energies of disc A;

1.0 point for the correct kinetic energies of disc B;

The total points of this part is 2.0