Theoretical Question 2

Sound Propagation

Introduction

The speed of propagation of sound in the ocean varies with depth, temperature and salinity. Figure 1(a) below shows the variation of sound speed c with depth z for a case where a minimum speed value c_0 occurs midway betweeen the ocean surface and the sea bed. Note that for convenience $z = 0$ at the depth of this sound speed minimum, $z = z_S$ at the surface and $z = -z_b$ at the sea bed. Above $z = 0$, c is given by

$$c = c_0 + bz \quad .$$

Below $z = 0$, c is given by

$$c = c_0 - bz \quad .$$

In each case $b = \left|\frac{dc}{dz}\right|$, that is, b is the magnitude of the sound speed gradient with depth; b is assumed constant.

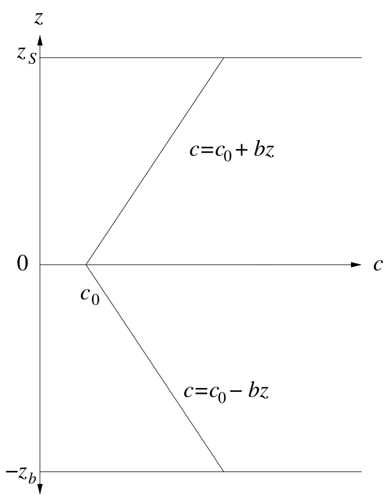

Figure 1 (a)

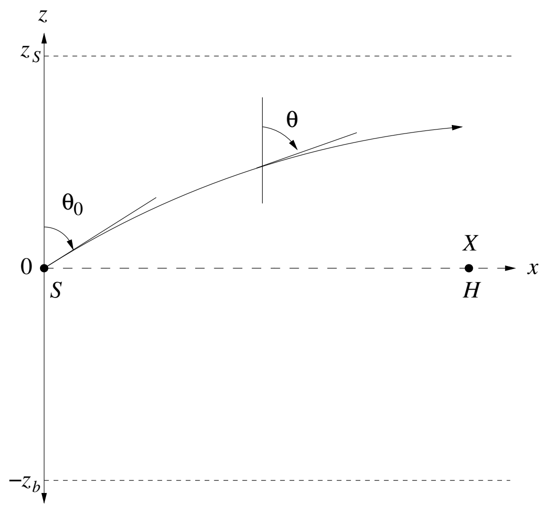

Figure 1 (b)

Figure 1(b) shows a section of the z–x plane through the ocean, where x is a horizontal direction. The variation of c with respect to z is shown in figure 1(a). At the position $z = 0$, $x = 0$, a sound source S is located. A 'sound ray' is emitted from S at an angle θ_0 as shown. Because of the variation of c with z, the ray will be refracted.

(a) (6 marks)
Show that the trajectory of the ray, leaving the source S and constrained to the z–x plane forms an arc of a circle with radius R where

$$R = \frac{c_0}{b \sin\theta_0} \quad \text{for} \quad 0 \leq \theta_0 < \frac{\pi}{2} \quad .$$

(b) (3 marks)
Derive an expression involving z_S, c_0 and b to give the smallest value of the angle θ_0 for upwardly directed rays which can be transmitted without the sound wave reflecting from the sea surface.

(c) (4 marks)
Figure 1(b) shows the position of a sound receiver H which is located at the position $z = 0$, $x = X$. Derive an expression involving b, X and c_0 to give the series of angles θ_0 required for the sound ray emerging from S to reach the receiver H. Assume that z_S and z_b are sufficiently large to remove the possibility of reflection from sea surface or sea bed.

(d) (2 marks)
Calculate the smallest four values of θ_0 for refracted rays from S to reach H when

- $X = 10000$ m
- $c_0 = 1500$ ms^{-1}
- $b = 0.02000$ s^{-1}

(e) (5 marks)
Derive an expression to give the time taken for sound to travel from S to H following the ray path associated with the **smallest** value of angle θ_0, as determined in part (c). Calculate the value of this transit time for the conditions given in part (d). The following result may be of assistance:

$$\int \frac{dx}{\sin x} = \ln \tan\left(\frac{x}{2}\right)$$

Calculate the time taken for the direct ray to travel from S to H along $z = 0$. Which of the two rys will arrive first, the ray for which $\theta_0 = \pi/2$, or the ray with the smallest value of θ_0 as calculated for part (d)?