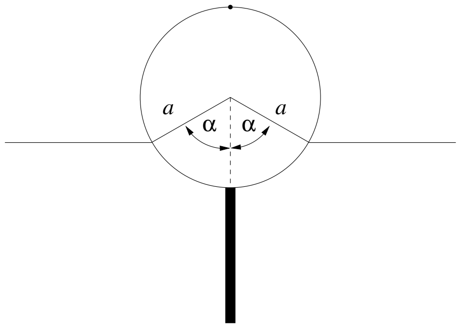
Theoretical Question 3

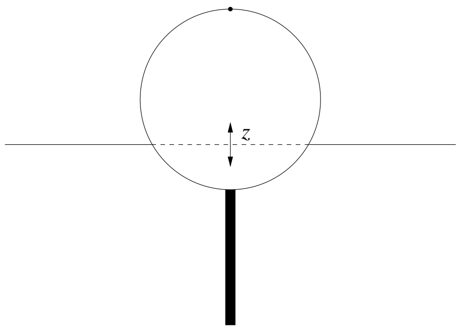
Cylindrical Buoy

(a) (3 marks)
A buoy consists of a solid cylinder, radius a, length l, made of lightweight material of uniform density d with a uniform rigid rod protruding directly outwards from the bottom halfway along the length. The mass of the rod is equal to that of the cylinder, its length is the same as the diameter of the cylinder and the density of the rod is greater than that of seawater. This buoy is floating in sea-water of density ρ.

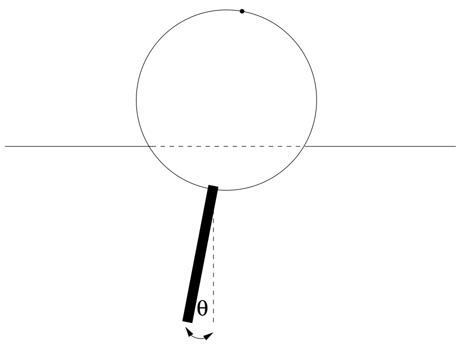
In equilibrium derive an expression relating the floating angle α, as drawn, to d/ρ. Neglect the volume of the rod.



(b) (4 marks)
If the buoy, due to some perturbation, is depressed vertically by a small amount z, it will experience a nett force, which will cause it to begin oscillating vertically about the equilibtium floating position. Determine the frequencty of this vertical mode of vibration in terms of α, g and a, where g is the acceleration due to gravity. Assume the influence of water motion on the dynamics of the buoy is such as to increase the effective mass of the buoy by a factor of one third. You may assume that α is not small.



(c) (8 marks)
In the approximation that the cylinder swings about its horizontal central axis, determine the frequency of swing again in terms of g and a. Neglect the dynamics and viscosity of the water in this case. The angle of swing is assumed to be small.



(d) (5 marks)
The buoy contains sensitive acelerometers which can measure the vertical and swinging motions and can relay this information by radio to shore. In relatively calm waters it is recorded that the vertical oscillation period is about 1 second and the swinging oscillation period is about 1.5 seconds. From this information, show that the floating angle α is about 90° and thereby estimate the radius of the buoy and its total mass, given that the cylinder length l equals a.

[You may take it that $\rho \simeq 1000\ \mathrm{kgm}^{-3}$ and $g \simeq 9.8\ \mathrm{ms}^{-2}$.]