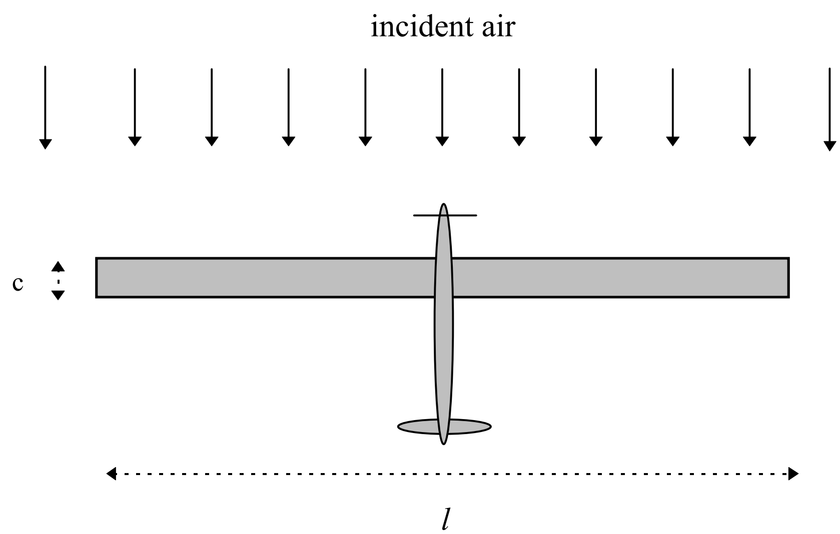
Theory Question No.3

Solar-Powered Aircraft

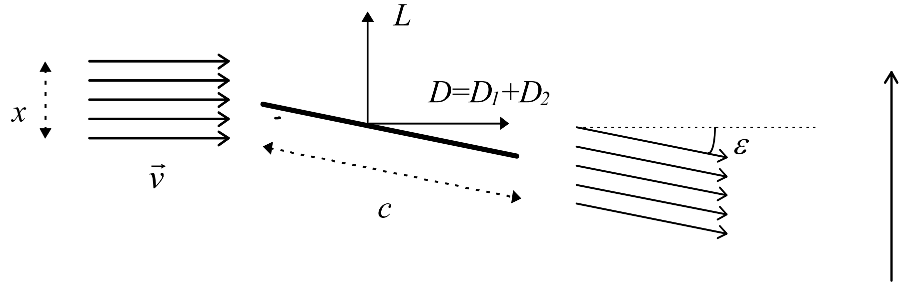
We wish to design an aircraft which will stay aloft using solar power alone. The most efficient type of layout is one with a wing whose top surface is completely covered in solar cells. The cells supply electrical power with which the motor drives the propeller.

Consider a wing of rectangular plan-form with span l, chord (width) c; the wing area is $S = cl$, and the wing aspect ratio $A = l / c$. We can get an approximate idea of the wing's performance by considering a slice of air of height x and length l being deflected downward at a small angle ε with only a very small change in speed. Control surfaces can be used to select an optimal value of ε for flight. This simple model corresponds closely to reality if $x = \pi l /4$, and we can assume this to be the case. The total mass of the aircraft is M and it flies horizontally with velocity $\vec{v}$ relative to the surrounding air. In the following calculations consider only the air flow around the wing.

Top view of aircraft (in its own frame of reference):



Side view of wing (in a frame of reference moving with the aircraft):



incident air wing section air leaving wing vertical(up)

Ignore the modification of the airflow due to the propeller.

(a) Consider the change in momentum of the air moving past the wing, with *no* change in speed while it does so. Derive expressions for the vertical lift force L and the horizontal drag force D_1 on the wing in terms of wing dimensions, v, ε, and the air density ρ. Assume the direction of air flow is always parallel to the plane of the side-view diagram. (*3 marks*)

(b) There is an additional horizontal drag force D_2 caused by the friction of air flowing over the surface of the wing. The air slows slightly, with a change of speed Δv (<< 1% of v) given by:

$$\frac{\Delta v}{v} = \frac{f}{A}$$

The value of f is independent of ε.

Find an expression (in terms of M, f, A, S, ρ and g- the acceleration due to gravity) for the flight speed v_0 corresponding to a minimum power being needed to maintain this aircraft in flight at constant altitude and velocity. Neglect terms of order ($\varepsilon^2 f$) or higher. (*3 marks*)

You may find the following small angle approximation useful:

$$1 - \cos\varepsilon \approx \frac{\sin^2\varepsilon}{2}$$

(c) On the answer sheet, sketch a graph of power P versus flight speed v . Show the separate contributions to the power needed from the two sources of drag. Find an expression (in terms of M, f, A, S, ρ and g) for the minimum power, P_{min}. (*2 marks*)

(d) If the solar cells can supply sufficient energy so that the electric motors and propellers generate mechanical power of $I = 10$ watts per square metre of wing area, calculate the maximum wing loading Mg/S (N/m^2) for this power and flight speed v_0 (m/s). Assume $\rho = 1.25$ kg/m^3, $f = 0.004$, $A = 10$. (*2 marks*)

Question 3: Answer Sheet **STUDENT CODE:**____________________

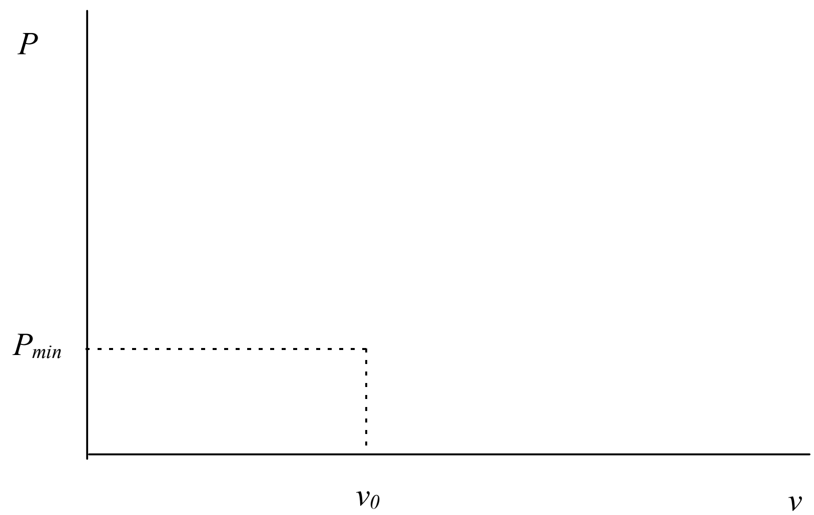
(a) Expression for L :

(a) Expression for D_1 :

(b) Expression for D_2 :

(b) Expression for v_0 :

(c)



(c) Expression for P_{min} :

(d) Maximum value of Mg/S :

(d) Numerical value of v_0 :