

Question Two ~ Solution

- (a) Focusing occurs for one "cyclotron" orbit of the electron.

Angular velocity $\omega = e B / m$; so time for one orbit $T = 2 \pi m / e B$

Speed of electron $u = (2 e V / m)^{1/2}$

Distance travelled $D = T u \cos \beta \approx T u = (2^{3/2} \pi / B) (V m / e)^{1/2}$

Thus charge to mass ratio = $e / m = 8 V \times (\pi / B D)^2$

- (b) Consider condition (ii) - Force due to electric field acts upwards

In region A force due magnetic field acts upwards as well, electron hits upper plate and does not reach the film.

In region B, force due magnetic field acts downwards, and *if* force is equal and opposite to the electrostatic force, there will be no unbalanced force, and electron will emerge from plates to expose film.

Piece was taken from region B.

- (c) We require forces to balance. Electric force given by eV / t , magnitude of magnetic force given by $e u B \sin \phi$, with u the speed of the electron.

For these to balance we require $u = V / t B \sin \phi$

Maximum u corresponds to minimum ϕ - at 23°

Therefore $u = 2.687 \times 10^8 \text{ m/s} = 0.896 c$.

Relativistic $\gamma = (1 - v^2/c^2)^{-1/2} = 2.255$,

so kinetic energy of electron = $(\gamma - 1) m c^2 = 641 \text{ keV}$.

- (d) After emerging from region between plates, electrons experience force due to magnetic field only. We approximate this by a vertical force, because angle of electron to horizontal remains small.

Acceleration caused by this force $a = B e u \sin \phi / \gamma m$

Initial horizontal speed is u , therefore time taken to reach the film after emerging from the region between the plates $t = s / u$.

Change in vertical displacement during this time $= y / 2 = \frac{1}{2} a (s / u)^2$

$$y = B e s^2 \sin \phi / \gamma m u$$

From part (f), for electron to have emerged from plate, we also know $u = V / t B \sin \phi$.

Therefore we eliminate u to obtain:

$$y^2 = (e B s \sin \phi / m)^2 \{ (B s t \sin \phi / V)^2 - (s / c)^2 \}$$

and we plot

VERTICAL	$(y / B s \sin \phi)^2$
HORIZONTAL	$(B s t \sin \phi / V)^2$

Therefore we have a gradient $(e / m)^2$

and a vertical-axis intercept $-(e s / m c)^2$

The intercept is read as $-537.7 (C s / kg)^2$, giving $e/m = 1.70 \times 10^{11} C / kg$

The gradient is read as $2.826 \times 10^{22} (C/kg)^2$, giving $e/m = 1.68 \times 10^{11} C / kg$.