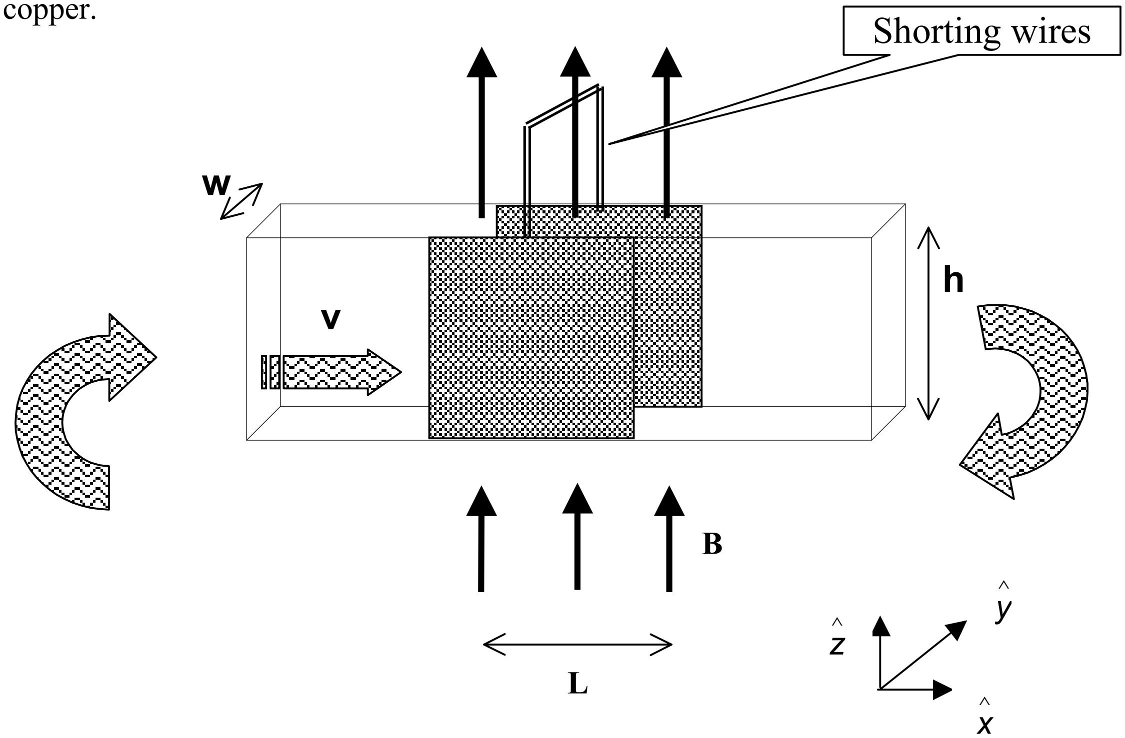
Question 3

MAGNETOHYDRODYNAMIC (MHD) GENERATOR

A horizontal rectangular plastic pipe of width w and height h, which closes upon itself, is filled with mercury of resistivity ρ. An overpressure P is produced by a turbine which drives this fluid with a constant speed v_0. The two opposite vertical walls of a section of the pipe with length L are made of copper.



The motion of a real fluid is very complex. To simplify the situation we assume the following:

- Although the fluid is viscous, its speed is uniform over the entire cross section.
- The speed of the fluid is always proportional to the net external force acting upon it.
- The fluid is incompressible.

These walls are electrically shorted externally and a uniform, magnetic field **B** is applied vertically upward only in this section. The set up is illustrated in the figure above, with the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ to be used in the solution.

a) Find the force acting on the fluid due to the magnetic field (in terms of L, B, h, w, ρ and the new velocity v) *[2.0 pts]*
b) Derive an expression for the new speed v of the fluid (in terms of v_0, P, L, B and ρ) after the magnetic field is applied. *[3.0 pts]*
c) Derive an expression for the additional power that must be supplied by the turbine to increase the speed to its original value v_0. Copy your result onto the **answer form.** *[2.0 pts]*
d) Now the magnetic field is turned off and mercury is replaced by water flowing with speed v_0. An electromagnetic wave with a single frequency is sent along the section with length L in the direction of the flow. The refractive index of water is n, and $v_0 \ll c$. Derive an expression for the contribution of the fluid's motion to the phase difference between the waves entering and leaving section L. Copy your result onto the **answer form.** *[3.0 pts]*