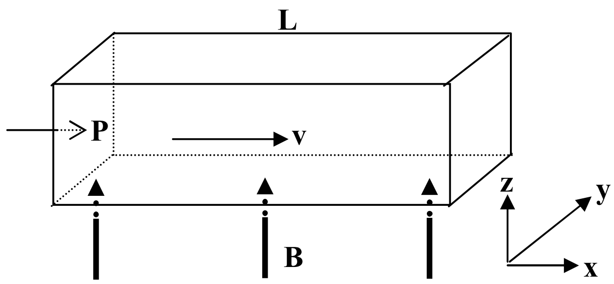
Part 3a



The net force on a charged particle must be zero in the steady state

$$\vec{F} = 0 = q\vec{E} + q\vec{v}x\vec{B}$$

$$\vec{E} = -\vec{v}x\vec{B} = vB\,\hat{y}$$ (0.4 pts)

$$V_H = vBw$$

$$I = \frac{V_H}{R} = \frac{V_H}{\frac{\rho w}{Lh}} = \frac{vBwLh}{\rho w} = \frac{vBLh}{\rho}$$, direction: **-** $\hat{y}$ (0.6 pts)

$$\vec{F} = I\vec{\ell}x\vec{B} = \frac{vB^2Lhw}{\rho}$$, direction: **(-** $\hat{y}\,x\,\hat{z} = -\hat{x}$ **)**

Force is in the -x direction (0.8 pts)

This creates a back pressure P_b

$$P_b = \frac{vB^2Lhw}{\rho hw} = \frac{vB^2L}{\rho}$$ (0.6 pts)

F_{net}=(P-P_b)hw, (0.6 pts)

v=αF_{net} (0.4 pts)

v=α(P-P_b)hw= $\alpha\,(P - \frac{vB^2L}{\rho})\frac{v_0}{\alpha P} = v_0 - \frac{vv_0B^2L}{P\rho}$

$$v(1 + \frac{v_0B^2L}{P\rho}) = v_0$$

$$v = v_0\left(1 + \frac{v_0B^2L}{P\rho}\right)^{-1}$$

$$v = v_0\,\frac{P\rho}{P\rho + v_0B^2L}$$ (0.6 pts)

Part 3b

From conservation of energy:

$$\Delta Power = V_H I = \frac{v_0^2 B^2 whL}{\rho}$$

or, **(1.0 pts)**

to recover v_0 the pump must supply an additional pressure $\Delta P = P_b$

$$\Delta Power = \Delta P hwv_0 = P_b hwv_0 = \frac{v_0^2 B^2 whL}{\rho}$$

Part 3c

1. $u = \frac{c}{n}$ $\qquad u' = \frac{\frac{c}{n} + v}{1 + \frac{c}{n}\frac{v}{c^2}} = \frac{\frac{c}{n} + v}{1 + \frac{v}{cn}}$ (0.5 pts)

   For small v (v<<c);

   neglect the terms containing $\frac{v^2}{c^2}$ in the expansion of $(1 + \frac{v}{cn})^{-1}$

   $$u' = (\frac{c}{n} + v)\frac{1}{1 + \frac{v}{cn}} \approx (\frac{c}{n} + v)(1 - \frac{v}{cn}) \approx \frac{c}{n} + v(1 - \frac{1}{n^2})$$

   $\Delta u = u' - u \approx v(1 - \frac{1}{n^2})$ (0.5 pts)

   $\Delta\phi = 2\pi f \Delta T$, $T = \frac{L}{u}$, $\Delta \mathrm{T} = \frac{\Delta \mathrm{u}}{\mathrm{u}^2} L \approx \frac{Lv}{c^2}(n^2 - 1)$ (0.5 pts)

   $v = v_0$ so that, $\Delta\phi = 2\pi f \frac{L}{c^2}(n^2 - 1)v_0$ (0.5 pts)

2. $\Delta\phi = 2\pi f \frac{L}{c^2}(n^2 - 1)v_0$

   a phase of $\pi/36$ results in (0.4 pts)

   $v_0 = \frac{c^2}{72L(n^2 - 1)f}$ (0.2 pts)

   $v_0 = \frac{9x10^{16}}{72x10^{-1}x(2.56 - 1)x25} = 3.2x10^{14}\, m/s$ which is not physical. (0.4 pts)

3. For v=20 m/s, f≈$4x10^{14}$ Hz. But for this value of f, skin depth is about 25 nm. This means that amplitude of the signal reaching the end of the tube is practically zero. Therefore mercury should be replaced with water. (0.6 pts)

On the other hand if water is used instead of mercury, at 25 Hz $\delta \approx 3x10^{5}$ m. Signal reaches to the end but v≈$6x10^{14}$ m/s, is still nonphysical. Therefore frequency should be readjusted. (0.6 pts)

For v=20 m/s electromagnetic wave of f≈$8x10^{14}$ Hz has a skin depth of about $\delta \approx 5.6$ cm in water and the emerging wave is out of phase by $\pi/36$ with respect to the incident wave. (The amplitude of the wave reaching to the end of the section is about 17% of the incident amplitude). (0.6 pts)

Therefore mercury should be replaced with water and frequency should be adjusted to f≈$8x10^{14}$ Hz. The correct choice is (iii) (0.2 pts)